Statistical Mechanics (M1)

Fermi-Dirac statistics

1 The 3D ideal fermion gas

Consider a 3D gas of N fermions in the thermodynamic limit.

a) Obtain the relationship between the grand potential J and the energy E of the gas. Derive the state equation of the gas.

b) Calculate the chemical potential (called Fermi level ε_F) and the interval energy of an ideal gas of fermions of spin $s = 1/2$, at $T = 0$. Compare with the classical case.

c) Calculate the chemical potential μ and the internal energy of an ideal gas of fermions of spin $s = 1/2$, at the lowest order in T, in the case of low but finite temperature.

d) Calculate the specific heat, the entropy, the free energy and the pressure of the gas at low temperature.

e) Calculate the pressure and the specific heat in the high temperature limit.

2 A system with a constant density of states

Let us consider a system of N independent electrons. In 2D, the corresponding density of states is

$$\rho(\varepsilon) = \begin{cases} D & \text{for } \varepsilon > 0 \\ 0 & \text{for } \varepsilon < 0 \end{cases}$$

where D is a constant.

a) Calculate the Fermi level.

b) Give the condition for the system to be non-degenerate.

c) Show that the specific heat is proportional to T for a highly degenerate system.

3 Pressure and compressibility of Sodium

a) Obtain the expression of the pressure for a highly degenerate gas.

b) Calculate the electronic compressibility K, defined by $K = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$ for a sodium crystal (which has only one free electron per atom).

Data : atomic mass of Na = 23g, density $\rho_{Na} = 0.97 g/cm^3$, Fermi temperature $T_F = 104K$, electron mass $m_e = 9.1 \cdot 10^{-31} kg$.