Exercises : First passage time and exit probability

1

Consider a 1D random walker (RW). At $t = 0$ place the RW at $x = 10$. Compute the distribution of times at which the walker arrives at the origin for the first time and plot it. What is the probability the RW arrives at $x = 0$ for the first time between t and $t + dt$? Compare with the theory.

2

Consider a discrete in time and space 1D RW such that the probability of performing a step to the right is p and to the left q, with $p = q = 1/2$. Place the walker at the origin at $t = 0$. Compute the distribution of times at which the walker returns to the origin and plot it. What is the probability the RW returns at any time $t > 0$ to the origin. Repeat the same problem for i) a 2D RW, where the RW can move up, down, left, and right with probability 1/4 and ii) a 3D walker where now the waker has 6 possible directions with probability 1/6. For i) and ii) compute the distribution of returning times as well as the probability of coming back to the origin at any time. Compare with the theory.

3

Consider a symmetric 1D RW characterized by a diffusion coefficient D. The walker moves on plateau of width W as shown in the figure. Assume that at $t = 0$ the walker is at the middle of the plateau. How long will the RW survive before falling off ? Is this time a function of W ? What is the distribution of these times ? Address the problem numerically and analytically.

4

Consider two symmetric 1D RW characterized by a diffusion coefficient D. The two RW are separated initially a distance d. Our RW have killing instincts and whenever they meet one will kill the other one instantaneously. For how long will we observe two RW in the system ? Find the distribution of these times and the dependency of these time on D and d.