Exercises : Anomalous diffusion

1

Consider a 1D anomalous walker discrete in time and assume that at each time step the walker makes a step Δ, where Δ is drawn from a distribution $p(\Delta) \propto \Delta^{-(1+\mu)}$ for $|\Delta| > 1$ and 0 otherwise (with probability 1/2, $\Delta > 0$ and 1/2 $\Delta < 0$) . Simulate two cases, $\mu = 1.5$ and $\mu = 2$, and estimate $\langle z_n \rangle$ and $MSD = \langle z_n^2 - \langle z_n \rangle^2 \rangle$, for each of them. Compare the numerical results with the theory. Plot $\langle z_n \rangle$ and MSD as function of the time : numerical simulations and theory.

2

Consider a 1D anomalous walker anomalous in time that at each step advance a distance $|\Delta| = const$ (with probability 1/2, $\Delta > 0$ and 1/2 $\Delta < 0$). The waiting time between one step and the next, for instance i, is given by τ_i, where τ_i is distributed according to $p(\tau) = \tau^{-(1+\mu)}$ (for $\tau > 1$, otherwise 0). Simulate two cases, $\mu = 0.5$ and $\mu = 1$, and estimate $\langle z(t) \rangle$, with $z(t)$ the position of the particle at time t, as well as $MSD = \langle z(t)^2 - \langle z(t) \rangle^2 \rangle$. Compare the numerical results with the theory. Plot $\langle z(t) \rangle$ and MSD as function of the time t.