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Random Walk (RW), Biased RW, Correlated RW

1

Consider a 1D RW discrete in time and space with probability p_0 of making a step to the right and q_0 of making a step to the left. Assuming $p_0 = q_0 = 1/2$, simulate an ensemble of $M = 1000$ independent random walker starting at the origin for $N = 1000$ time steps. Plot the average position$\langle x(n) \rangle$, and the mean square displacement (MSD) $MSD(n) = \langle (x(n)^2 - \langle x(n) \rangle)^2 \rangle$, with n the time step such that $1 < n < N$. Compute numerically the associated diffusion coefficient D, as well as the distribution function $p(x, n)$ of finding a RW at position x at time n, with $n = 10$ and $n = 100$. Compare the obtained numerical results, i.e., $\langle x(n) \rangle$, MSD, D, and $p(x, n)$, with the theory.

2

Repeat the same procedure as in **1**, but with $p_0 = q_0 = 1/3$. Specialize the theory for $p_0 = q_0 = 1/3$ and derive expression for $\langle x(n) \rangle$, MSD, D, and $p(x, n)$. Compare theory and numerical simulations.

3

Consider a totally asymmetric RW discrete in time and space with probability $p_0 = 1/3$ (meaning $q_0 = 0$). Start with all walkers at $x = 0$. Simulate the system and plot $\langle x(n) \rangle$, MSD, and $p(x, n)$. Compare with the theory.

4

Consider a 1D RW discrete in time and space with probability $p_0 = 1/2$ of making a step to the right and $q_0 = 1/4$ of making a step to the left. Simulate an ensemble of $M = 1000$ independent random walker starting at the origin for $N = 1000$ time steps. Plot the average position$\langle x(n) \rangle$ and the mean square displacement MSD. Compare the obtained curves with the theory. Derive an approximate expression for $\partial_n p(x, n)$ from the expression for $p(x, n + \Delta_t)$ and $p(x + \Delta_x, n)$. Try to integrate in the the system of PDEs.

5

Consider a correlated 1D RW with internal state $+$ and $-$ such that

1. Initially all $M = 1000$ particles are located at the origin and half of them are in state $+$ while the other half in state $-$.
2. At every time step $\Delta t = 1$ particle in state $+$ make a step of width $\Delta x = 1$ to the right, while those in state $-$ make an step to the left of the same width.
3. At every time step, with probability $1 - \lambda \Delta t$, with $\lambda = 0.1$, particles remain in their state. With probability $\lambda \Delta t$ they switch states, i.e., $+ \to -$ and $- \to +$

Simulate the system described for 1000 time steps and plot $\langle x(n) \rangle$ and MSD. Compare with the theory.

6

Consider a biased correlated 1D RW with internal state $+$ and $-$ similar to the previous one (see **5**). This time, consider that the transition $+ \to -$ is characterized by $\lambda_+ = 0.1$ and while the transition $- \to +$ by $\lambda_- = 0.05$. Simulate the system for 1000 time steps and plot $\langle x(n) \rangle$ and MSD.

7

Consider a biased correlated 1D RW with internal state $+$ and $-$ similar to the previous one. Assume that $\lambda_+ = \lambda_- = 0.05$. This time assume that a particle in state $+$ performs a step to the right with probability per time step Δt given by $\beta_+ \Delta_t$, while a particle in state $-$ makes an step to the left with probability $\beta_- \Delta_t$. Let us assume $\beta_+ = 0.1$ and $\beta_- = 0.2$. Simulate the system for 1000 time steps and plot $\langle x(n) \rangle$ and MSD.