Dynamical Systems

Maps : fixed points, linear stability, and periodic orbits

1 Order and fixed points in maps

Consider the following dynamical system :

$$x_{n+1} = a - b\,x_{n-1} - x_n^2$$

where a and b are constants.

1. Express the map as a first order one.
2. Find the fixed points
3. Evaluate the linear stability of the fixed points

2 A population dynamic model

The size of a population follows the following dynamics :

$$N_{t+1} = N_t \exp[r(1 - N_t/k)]$$

where r and k are real and positive constants. Note that for values of N_t, the growth rate is given by $\exp[r]$, while for N_t large enough the growth saturates and can even decrease.

1. Find fixed points
2. Evaluate the linear instability of the fixed points
3. Study the asymptotic evolution of the system around $r = 2$.

3 The tent map

The tent maps is defined by :

$$x_{n+1} = f(x_n) = 1 - 2|x_n|$$

which is defined in the interval $[-1, 1]$.

1. Find fixed points.
2. Find orbits of period 2 and 3 ; recall that these satisfy $x_{n+m} = f_m(x_n)$, with $f_m(\cdot) = f(f(....))$.
3. Evaluate the stability of the m-periodic orbits.