

# Next generation neural mass models

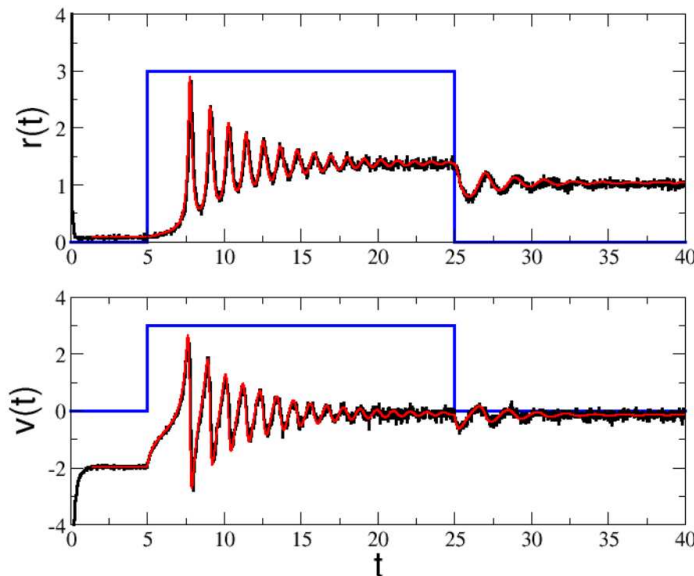
Alessandro Torcini

`alessandro.torcini@cyu.fr`

LPTM, CY Cergy Paris Université



# Plan of the Talk



- The Quadratic Integrate and Fire (QIF) Neuron
- Derivation of the Neural Mass Model (MPR) (Montbrió, Pazó, Roxin, PRX, 2015)
- Applications of the Neural Mass Model to Interacting Populations (2020)
  - Cross frequency coupling
  - Theta-nested gamma oscillations
  - Synaptic-based working memory
  - Spike frequency adaptation (2023)
- Extension of the neural mass to fluctuation driven population dynamics (2021)
  - Neural Networks with Background Noise
  - Neural Networks with Random Sparse Synaptic Couplings

# Quadratic Integrate and Fire Neuron

This a very simple model, which can mimic realistic neural dynamics

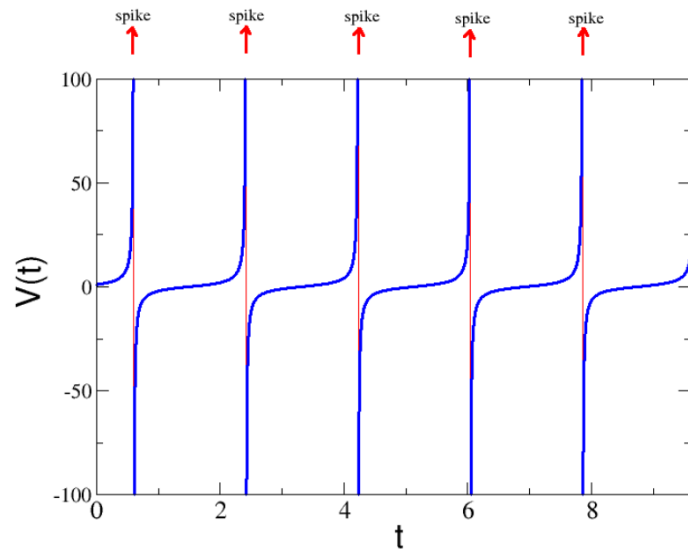
$$\frac{dV}{dt} = V^2 + \eta$$

- $V$  membrane potential with threshold  $V_{th}$  and reset  $V_r$
- $\eta$  neural excitability
  - $\eta > 0$  Tonic Neuron
  - $\eta \leq 0$  Excitable Neuron

The model has been developed to reproduce **parabolic bursting** in neurons of the Aplysia abdominal ganglion (when sinusoidally forced) and **low firing neurons** (Class I).

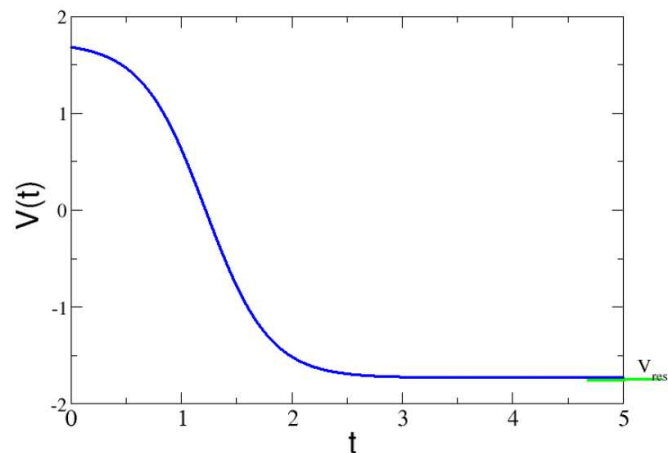
Ermentrout and Kopell, (1986) SIAM Journal on Applied Mathematics  
Latham et al. (2000) Journal of Neurophysiology

# Quadratic Integrate and Fire Neuron



Tonic neuron  $\eta > 0$

- The firing period is  $T_{firing} = \frac{\pi}{\sqrt{\eta}}$



Excitable neuron  $\eta \leq 0$

- If  $V(t_0) \leq \sqrt{|\eta|}$  - Subthreshold Dynamics  $V \rightarrow V_{rest} = -\sqrt{|\eta|}$
- If  $V(t_0) > \sqrt{|\eta|}$  emission of a spike followed by relaxation to  $V_{rest}$

# Period Tonic QIF Neuron

$$\frac{dV}{dt} = V^2 + \eta \quad \eta > 0 \quad V \in (-\infty, +\infty)$$

The model can be solved for separation of variables for external DC current  $\eta = \text{const}$

$$T = \int_0^T dt = \int_{-\infty}^{+\infty} \frac{dV}{V^2 + \eta} = \frac{1}{\sqrt{\eta}} \left[ \arctan \left( \frac{V}{\sqrt{\eta}} \right) \right]_{-\infty}^{+\infty} = \frac{\pi}{\sqrt{\eta}}$$

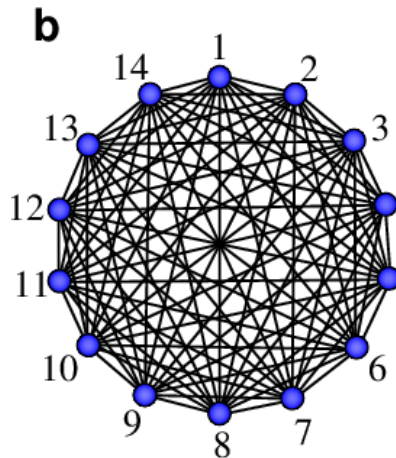
therefore the frequency is

$$\nu = \frac{1}{T} = \frac{\sqrt{\eta}}{\pi}$$

The frequency goes to zero for  $\eta \rightarrow 0$  : neuron of class I

This kind of dependence ( $\sqrt{\eta}$ ) is characteristic of Pyramidal neurons in the V layer of the cortex

# Population of QIF neurons



The evolution of the membrane potentials of  $N$  globally coupled **heterogeneous** neurons can be written as

$$\frac{dV_j}{dt} = V_j^2 + \eta_j + I_e(t) + Jr(t)$$

where

- $\eta_j$  is the excitability of neuron  $j$
- $I_e(t)$  is some external current
- $J$  is the synaptic coupling –  $J > 0$  ( $J < 0$ ) **excitatory** (**inhibitory**) neurons
- 

$$r(t) = \frac{1}{N} \sum_j \sum_{k(j)} \delta(t - t_{k(j)}) = \frac{1}{N} \sum_j r_j(t)$$

is the average firing rate due to all the post-synaptic potentials ( $\delta$ -spikes) emitted by all neurons in the network

# Neural Mass Model

In the limit  $N \rightarrow \infty$  we can describe the population of  $N$  neurons in term of a probability density function (PDF)  $\rho(V|\eta, t)$  of their membrane potentials

## Continuous formulation

$\rho(V|\eta, t)dV$  = fraction of neurons with membrane potentials between  $V$  and  $V + dV$ , with excitability  $\eta$ , at time  $t$

The excitabilities  $\eta_j \Rightarrow$  are continuous random variable distributed according to a PDF  $g(\eta)$

Since the number of neurons should be conserved in time, the PDF  $\rho$  satisfies the following

$$\text{Continuity equation} \quad \partial_t \rho + \partial_V [(V^2 + \eta + Jr + I_e)\rho] = 0$$

The **stationary solution**  $\rho_0$  (for constant  $I_e$ ) is given by

$$\rho_0(V|\eta) \propto \frac{1}{V^2 + \eta + Jr + I_e}$$

this is a **Lorentzian (or Cauchy) Distribution**  $L(x) = \frac{\Delta}{\pi(x^2 + \Delta^2)}$

# Neural Mass Model

## Lorentzian Ansatz

MPR assumed that also for non-stationary case the distribution is Lorentzian at any time

$$\rho(V|\eta, t) = \frac{1}{\pi} \frac{x(\eta, t)}{[V - y(\eta, t)]^2 + x(\eta, t)^2}$$

where the median  $y(\eta, t)$  and the HWHM  $x(\eta, t)$  correspond to the

Mean Membrane Potential

Firing Rate

$$v(\eta, t) = y(\eta, t)$$

$$r(\eta, t) = \frac{x(\eta, t)}{\pi}$$

for the neurons with excitability  $\eta$   
Namely,

for the neurons with excitability  $\eta$   
Namely,

$$v(\eta, t) = \lim_{R \rightarrow \infty} \int_{-R}^R \rho(V|\eta, t) V dV$$

$$r(\eta, t) = \lim_{V \rightarrow +\infty} \rho(V|\eta, t) \dot{V}$$



# Lorentzian and Ott-Antonsen Ansatz

The Fourier transform of a Lorentzian distribution distribution is

$$\mathcal{F}_V \left[ \frac{1}{\pi} \frac{\Delta}{[V - V_0]^2 + \Delta} \right] = e^{ikV_0 - \Delta|k|} = f_k$$

where  $V_0$  is the median of the LD and  $\Delta$  the HWHM of the LD.

Therefore one has that for  $k > 0$  that

$$f_k = (f_1)^k$$

the Lorentzian distribution satisfies the Ott-Antonsen Ansatz, let us say it represents a special case of it

Please notice that

$$\log f_k = ikV_0 - \Delta|k|$$

# Neural Mass Model

Therefore within the Lorentzian Ansatz the continuity equation can be rewritten as follows for the complex variable  $w(\eta, t) = \pi r(\eta, t) + iv(\eta, t)$

$$\partial_t w(\eta, t) = i[-w^2(\eta, t) + \eta + Js + I]$$

This is a neural mass equation describing the dynamics of a population of neurons with the same excitability  $\eta$  in terms of their firing rate  $r(\eta, t)$  and their mean membrane potential  $v(\eta, t)$ .

However for a heterogeneous population one has still an extremely large number of equations corresponding to all the possible values of the excitability  $\eta_i$

How can we reduce the system to a low dimensional one ?

# Neural Mass Model

Systems with **random heterogeneities** have been treated exactly in statistical mechanics by assuming a **Lorentzian distribution** for the heterogeneities E. Yakubovich, Soviet Physics JETP. 1969

$$g(\eta) = \frac{1}{\pi} \frac{\Delta}{(\eta - \bar{\eta})^2 + \Delta^2}$$

This allows to estimate **exactly** the average mean membrane potential and firing rate

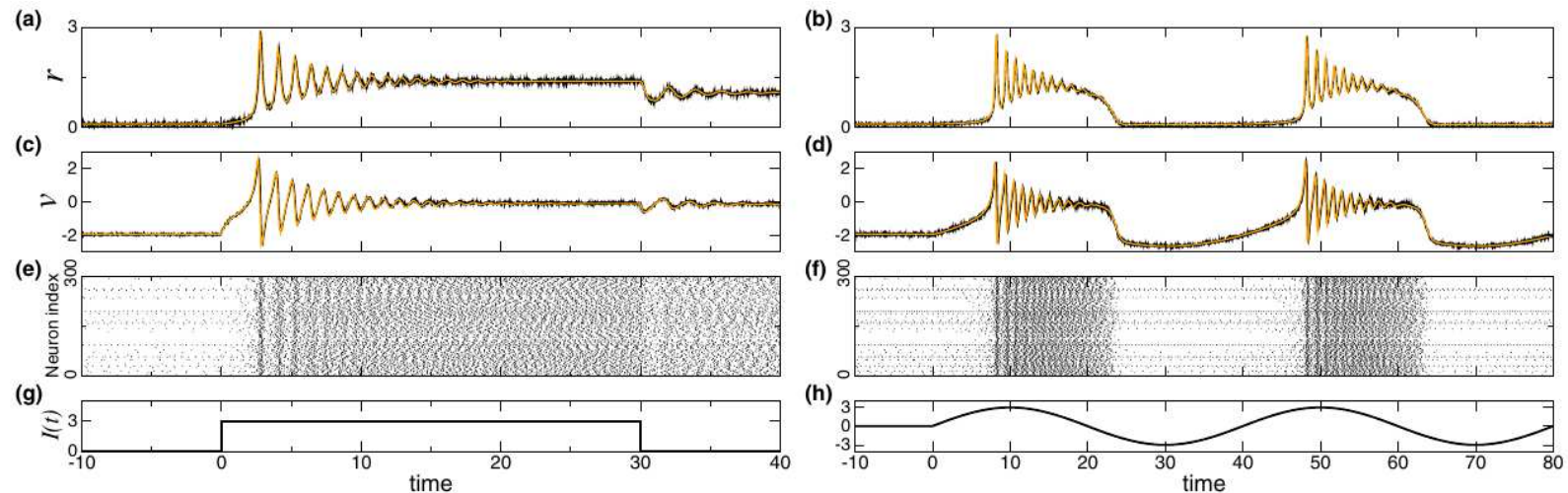
$$v(t) = \int_{-\infty}^{+\infty} v(\eta, t) g(\eta) d\eta \quad r(t) = \int_{-\infty}^{+\infty} v(\eta, t) g(\eta) d\eta$$

## 2-dimensional Neural Mass Model

$$\dot{r} = \frac{\Delta}{\pi} + 2rv \quad \dot{v} = v^2 + \bar{\eta} + Jr + I(t) - \pi^2 r^2$$

[Montbrió, Pazó, Roxin, Phys. Rev. X (2015)]

# MPR Model



## 2-dimensional Neural Mass Model

$$\dot{r} = \frac{\Delta}{\pi} + 2rv \quad \dot{v} = v^2 + \bar{\eta} + Jr + I(t) - \pi^2 r^2$$

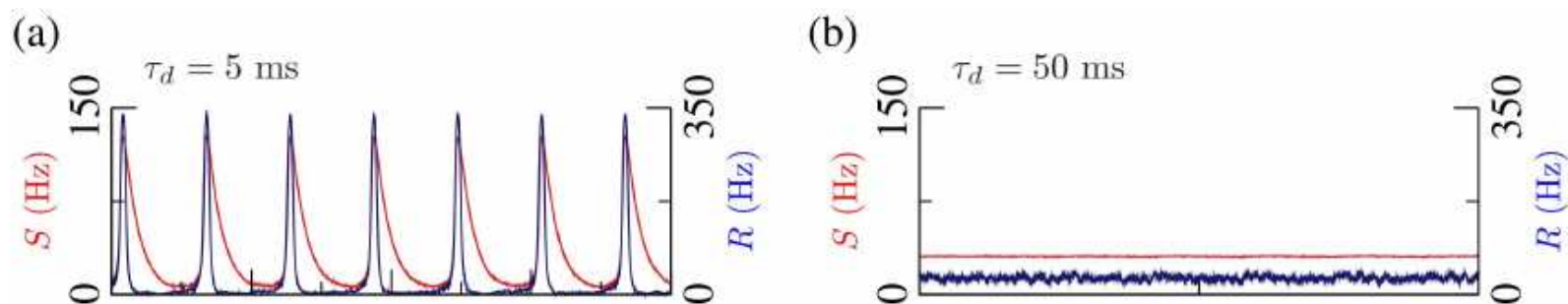
This Neural Mass Model describes **exactly** the dynamics of the Network done of  $N$  QIF neurons

[Montbrió, Pazó, Roxin, Phys. Rev. X (2015)]

# $\gamma$ Oscillations in Inhibitory Networks

Fast  $\gamma$  oscillations emerge in recurrently coupled inhibitory networks

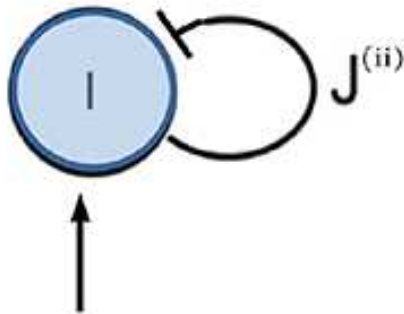
- An external **excitatory drive** tends to synchronize the inter-neurons that fire together
- The firing of the **inhibitory neurons** leads to their **silencing** on a timescale dictated by the duration  $\tau_d$  of the post-synaptic potentials  $S$
- This mechanism generates **collective oscillations** in the network  
[Whittington, Traub, Jefferys, Nature. 1995]



If  $\tau_d \gg \tau_m$ , the synaptic time scale is longer than the membrane time scale **no oscillations**, the recurrent input appears as an average inhibitory current

[DeValle, Roxin, Montbrió, PLOS Comp Biol (2017)]

# $\gamma$ Oscillations in Inhibitory Networks

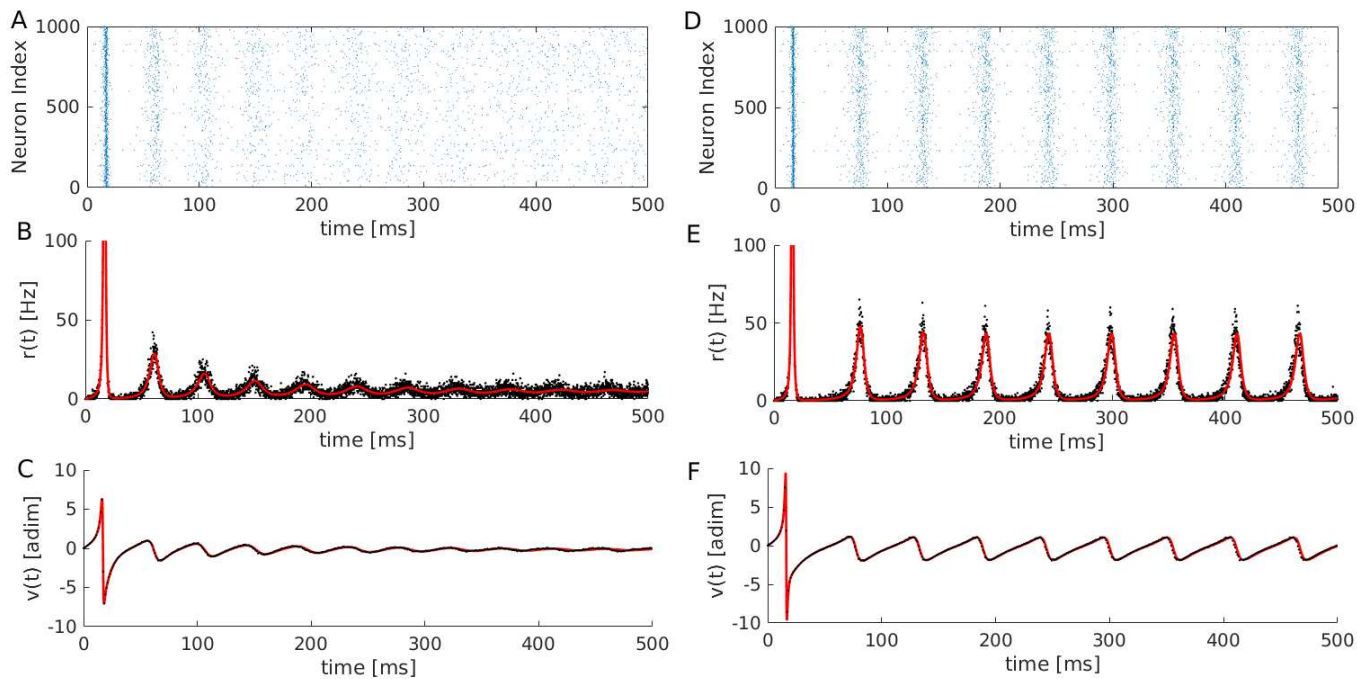


$$\tau_m \dot{r} = \frac{\Delta}{\tau_m \pi} + 2rv$$

$$\tau_m \dot{v} = v^2 + \bar{\eta} - \tau_m J^{(ii)} S + I^{(i)} - (\pi \tau_m r)^2$$

$$\tau_d \dot{S} = -S + r$$

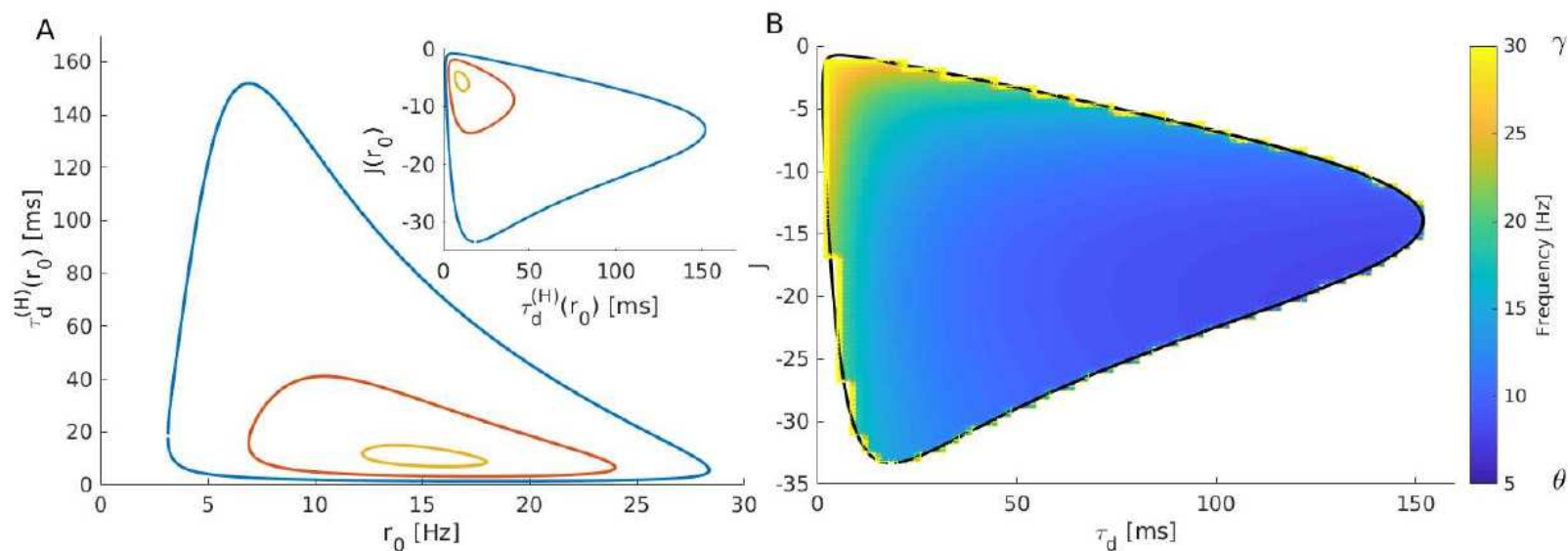
super-critical Hopf-Bifurcation to COs



Ceni, Olmi, AT, Angulo Garcia, Chaos (2020)

# $\gamma$ Oscillations in Inhibitory Networks

- Oscillations are sustained within a frequency range ( $\nu \simeq 5 - 30 \text{ Hz}$ ) thanks to
  - finite synaptic time  $\tau_d$
  - self-inhibitory action of neurons



- For increasing heterogeneity ( $\Delta = 0.01$ ,  $\Delta = 0.05$ ,  $\Delta = 0.1$ ,  $\Delta = 0.14$ ) the observation of collective oscillations requires finer tuning of the parameters
- Frequency decreases for increasing  $\tau_d$  and  $J$

# $\gamma$ Oscillations in Inhibitory Networks

The heuristic firing rate models (e.g. Wilson-Cowan model)

$$\begin{aligned}\tau_m \dot{r} &= -r + F(-\tau_m J^{(ii)} S + I^{(i)}) \\ \tau_d \dot{S} &= -S + r\end{aligned}$$

do not display **Collective Oscillations** without introducing **an effective delay** which takes somehow in account for the sub-threshold dynamics

- Firing rate models do not encompass Subthreshold Dynamics of the Membrane potential
- The membrane potential dynamics is fundamental to have synchronization effects
- The MPR model reproduces network dynamics in this case, Wilson-Cowan not

[DeValle, Roxin, Montbrió, PLOS Comp Biol (2017)]

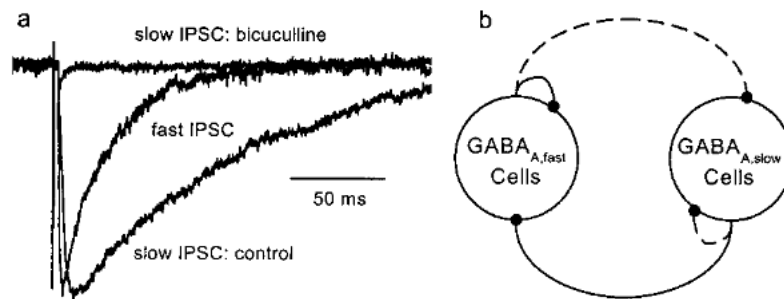


# $\theta$ - $\gamma$ Cross Frequency Coupling

One of the most present cross-frequency coupling in the brain is the interaction between **slow  $\theta$ -rhythms (5-10 Hz)** and **fast  $\gamma$ -oscillations (20-100 Hz)**

Which is the origin of this CFC ?

White, Banks, Pearce, and Kopell, PNAS 2000 proposed that  $\theta$ - $\gamma$  CFC emerges due to the interaction of **two inhibitory populations** with different kinetic properties



Two classes of interneurons identified in the Hippocampus (CA1)

- GABA<sub>A,fast</sub>  $\tau_d \simeq 9$  ms
- GABA<sub>A,slow</sub>  $\tau_d \simeq 50$  ms

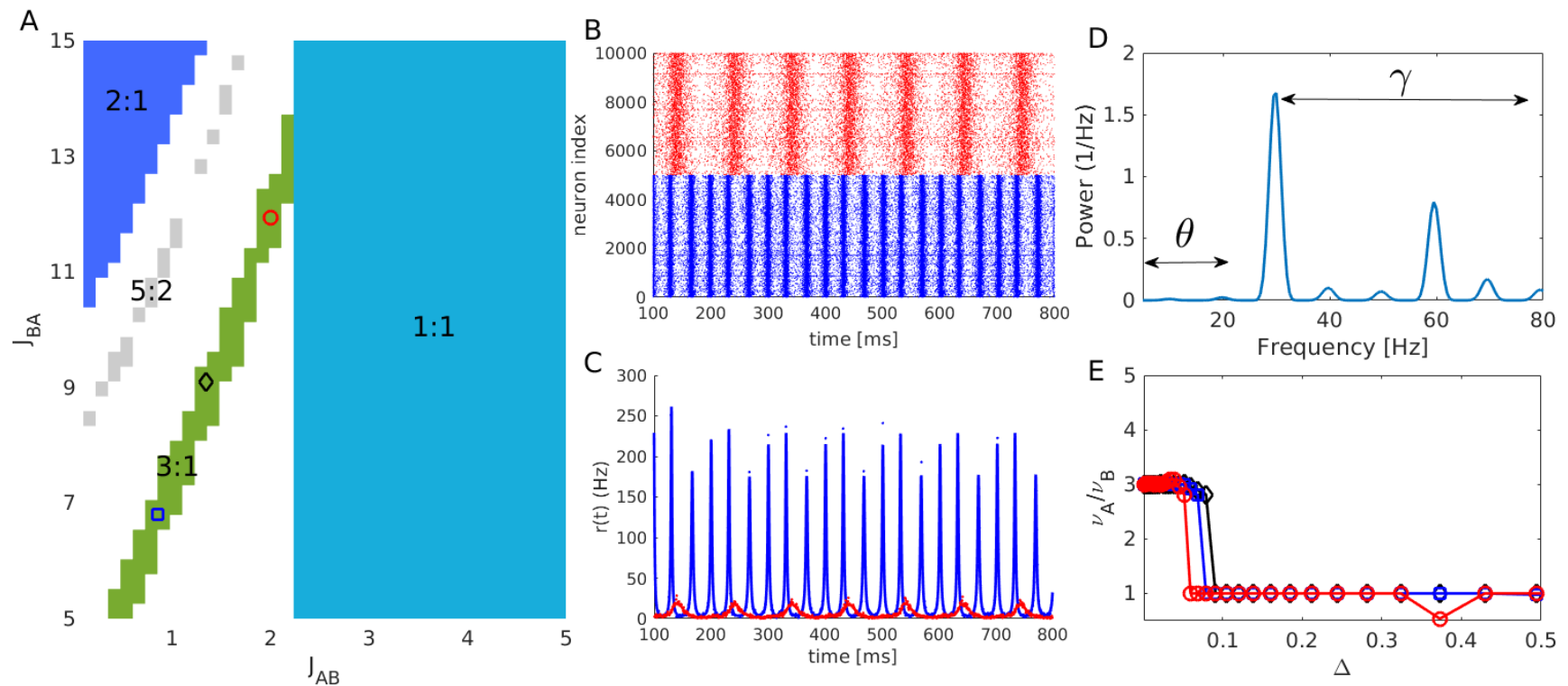
May we observe this phenomenon for two coupled neural masses  $A$  and  $B$  with exponential synapses with different kinetics ?

- A **slow inhibitory population A** with self-coupling  $J_{AA} = -2$ ,
- A **fast inhibitory population B** with self-coupling  $J_{BB} = -11$
- Cross-couplings  $J_{AB}$  ( $A \rightarrow B$ ) and  $J_{BA}$  ( $B \rightarrow A$ )

# $\theta$ - $\gamma$ Cross Frequency Coupling

Fast population:  $\tau_{A,d} = 9$  ms - Slow population:  $\tau_{B,d} = 50$  ms

3 : 1 phase-locking mode:  $\theta - \gamma$  coupling

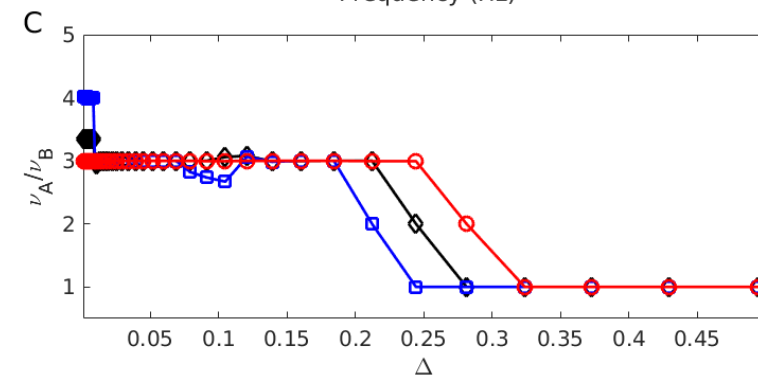
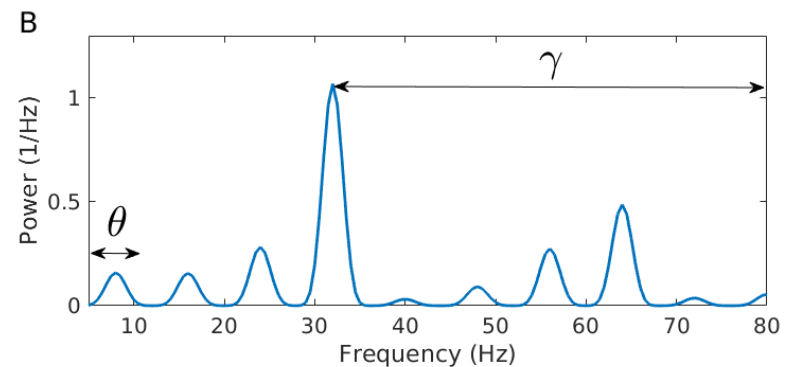
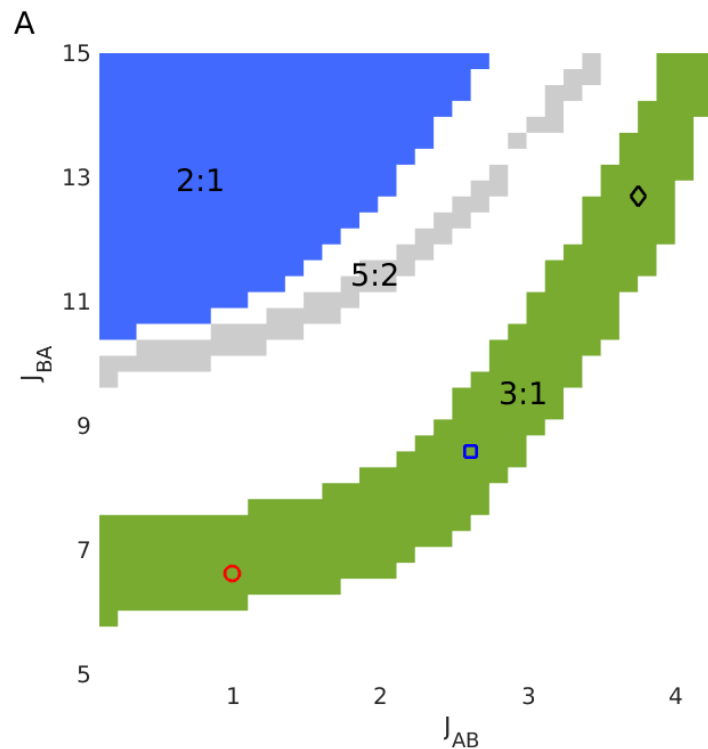


⇒ For large values of  $J_{AB}$  and disorder  $\Delta$ , the 3 : 1 locked mode is lost

[Ceni, Olmi, AT, Angulo Garcia, Chaos (2020)]

# $\theta$ - $\gamma$ Cross Frequency Coupling

- External  $\theta$ -forcing on the slow population  $\theta \simeq 10$  Hz
- Fast population:  $\tau_{A,d} = 9$  ms - Slow population:  $\tau_{B,d} = 50$  ms

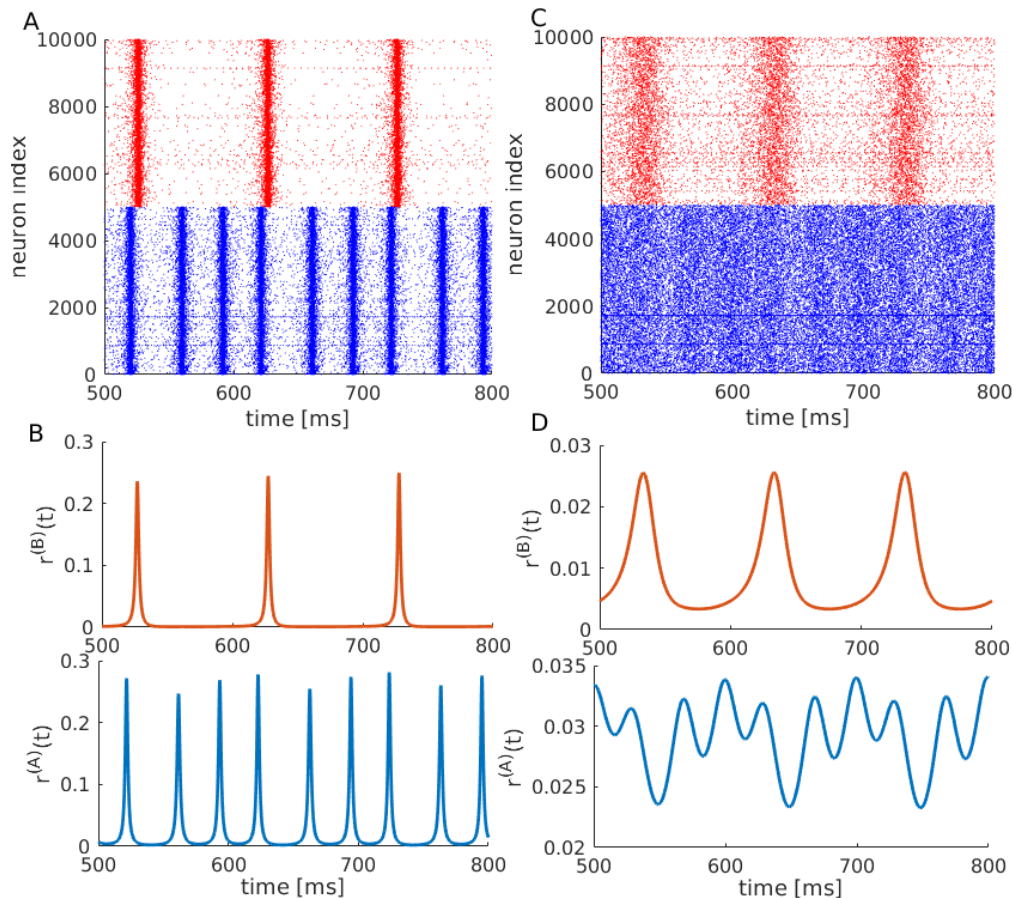


- ⇒ Power in the  $\theta$  band is increased
- ⇒ Adding the (slow) modulation to the slow population increases the amount of disorder ( $\Delta$ ) that can sustain the  $\theta - \gamma$  coupling

[Ceni, Olmi, AT, Angulo Garcia, Chaos (2020)]

# $\theta$ - $\gamma$ Cross Frequency Coupling

$$I_B = I_0 \sin(2\pi 10t)$$



■  $\Delta = 0.05$

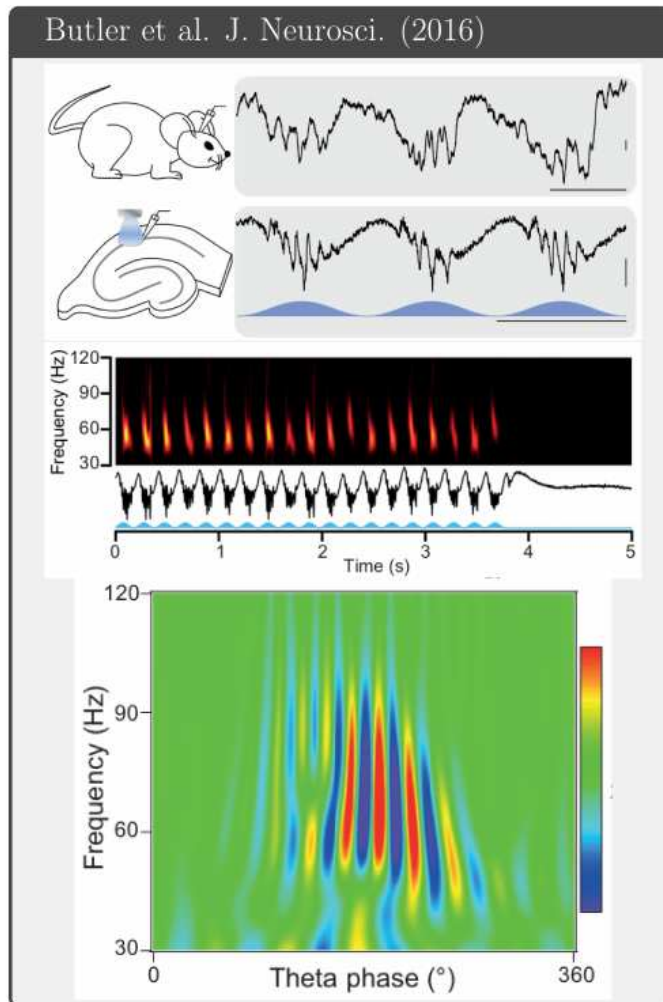
■ **Phase-phase CFC:**  
collective oscillations  
of the slow population  
and of the fast one are  
locked in phase

■  $\Delta = 0.2$

■ **Phase-amplitude CFC:**  
the slow population  
modulates the  
amplitude of the  
oscillations of the fast  
one

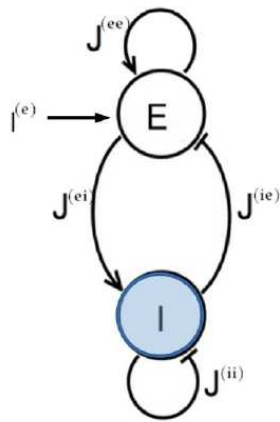
# $\theta$ Nested $\gamma$ Oscillations

Several **optogenetic experiments** performed in different areas of the hippocampus and entorhinal cortex suggest that a  $\theta$  frequency drive is sufficient to induce **in vitro**  $\theta$ - $\gamma$  CFCs



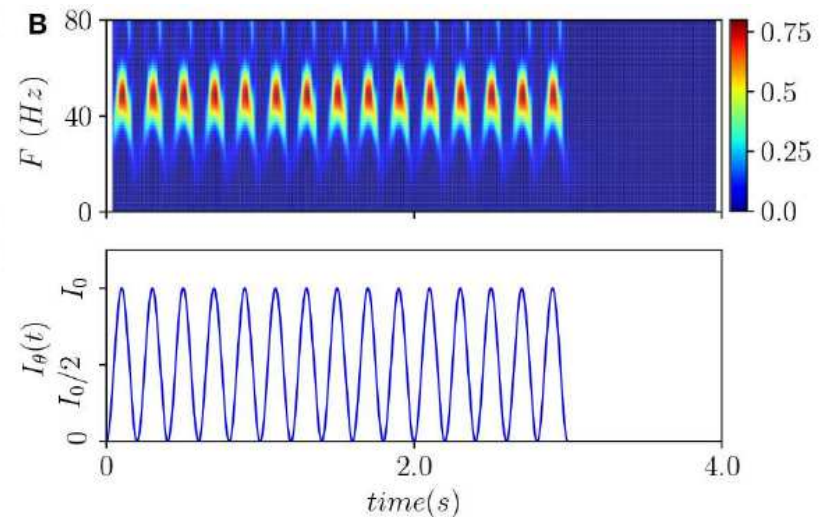
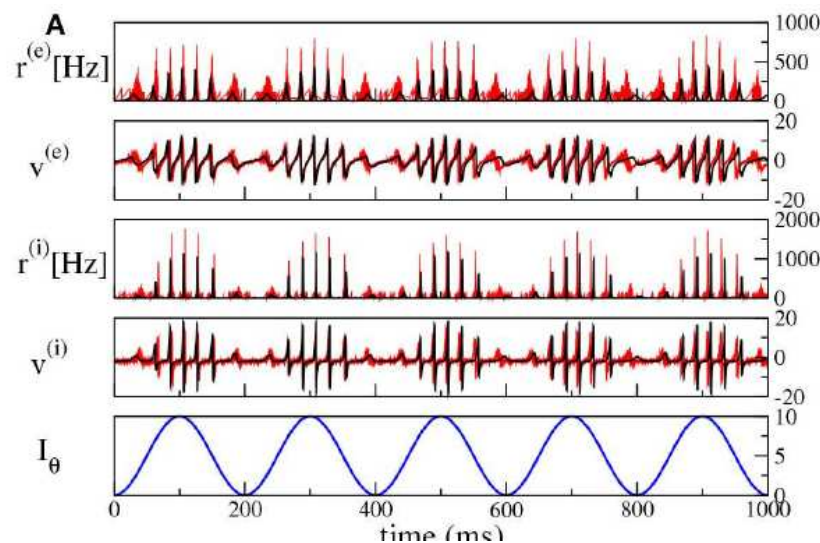
- Butler et al. (2016) stimulated CA1 pyramidal neurons with an excitatory drive  $I_{\theta} = I_0 \sin(\nu_{\theta} t)$  with  $\nu_{\theta} \in [1 : 10]$  Hz in vitro
- They show that this is sufficient to generate intrinsic CA1  $\gamma$  oscillations **in vitro** similar properties to **in vivo** CA1  $\gamma$  oscillations.
- They suggest that the mechanism for the generation of the oscillations is of the **pyramidal - interneuron gamma (PING)** type

# $\theta$ Nested $\gamma$ Oscillations



## Excitatory-Inhibitory Neural Masses

- We consider two coupled populations : an **excitatory** and an **inhibitory** plus an **external periodic forcing** on the excitatory population
- We observe  $\theta$  nested  $\gamma$  oscillations as in the experiments

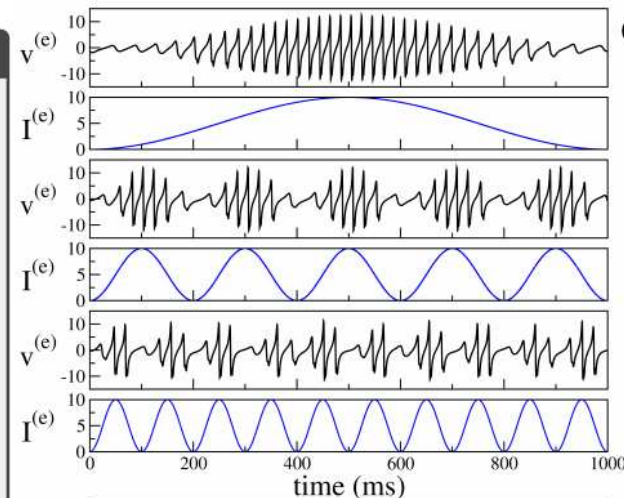
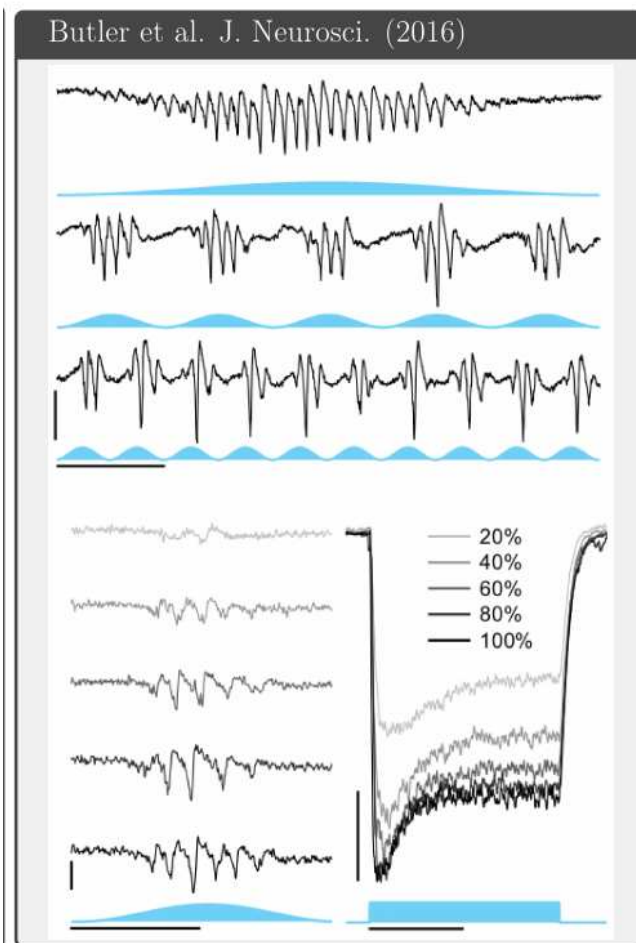


[Segneri, Bi, Olmi, AT Front. Comp. Neuroscience (2020)]

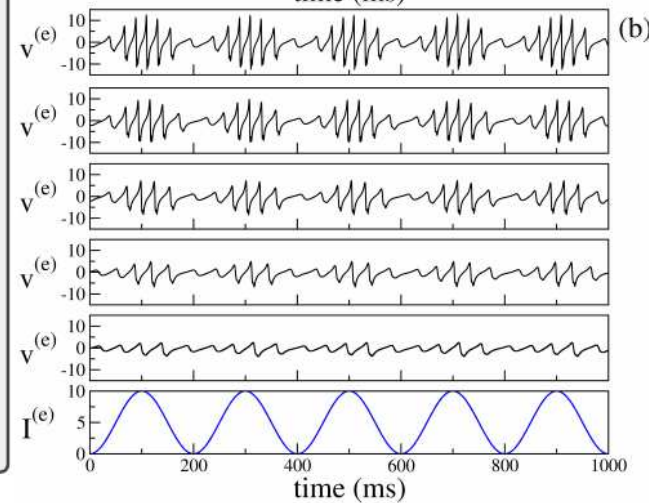
# $\theta$ Nested $\gamma$ Oscillations

Comparisons with the experiments

$$I^{(e)}(t) = I_0 \sin(\nu_\theta t)$$

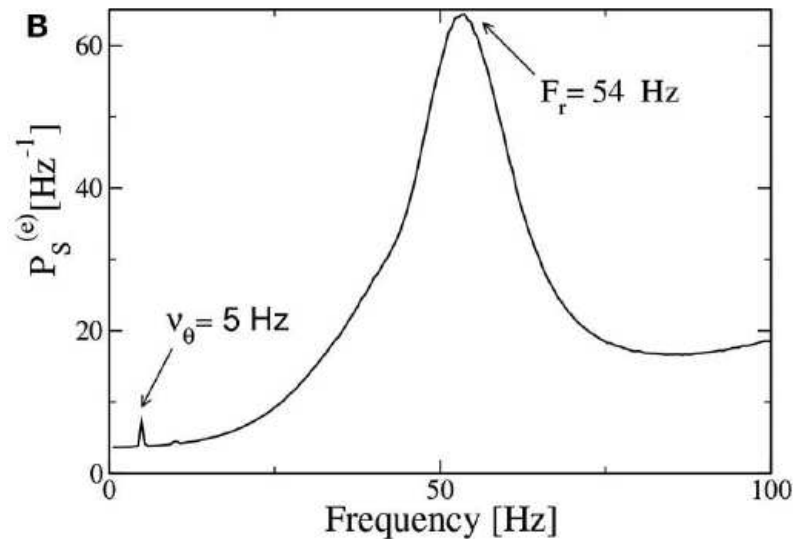


- nested  $\gamma$  oscillations are observable in the whole  $\theta$ -range  $\nu_\theta = [1 : 10]$  Hz
- For increasing  $I_0$  the amplitude of the  $\gamma$ -oscillations increases



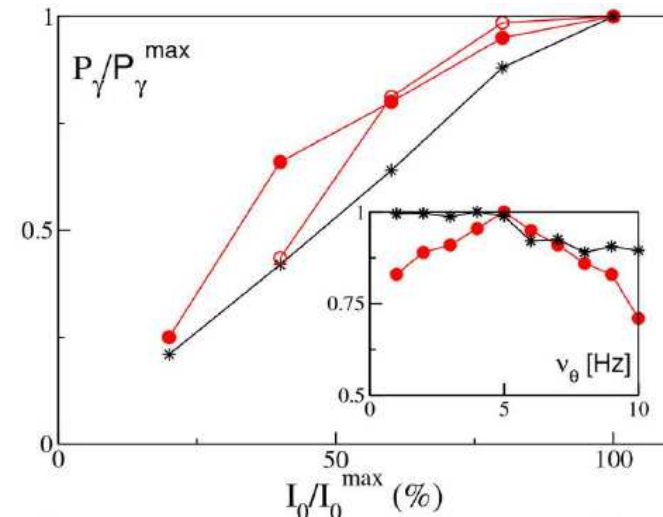
# $\theta$ Nested $\gamma$ Oscillations

## Power Spectra



- $\nu_{\theta}$  : forcing frequency
- $F_r$  : response frequency
- $I_0$  : forcing amplitude
- $P_{\gamma}$  : power under the main  $\gamma$ -peak

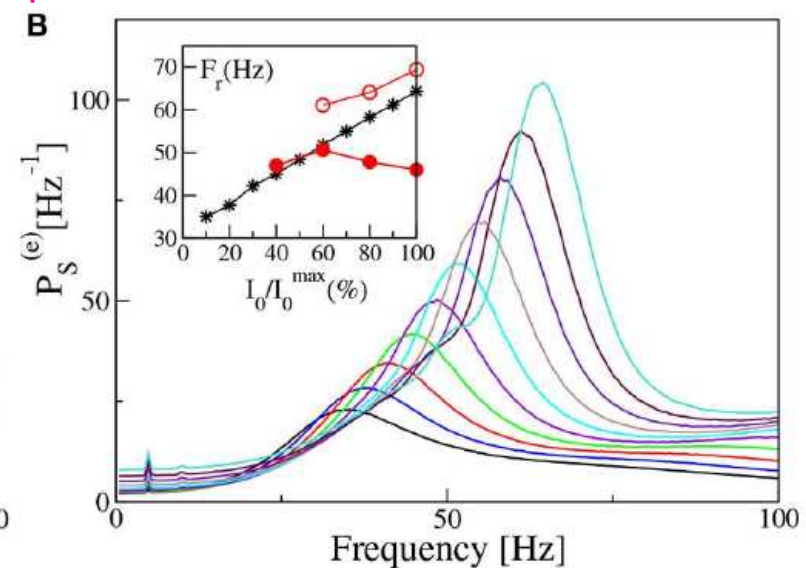
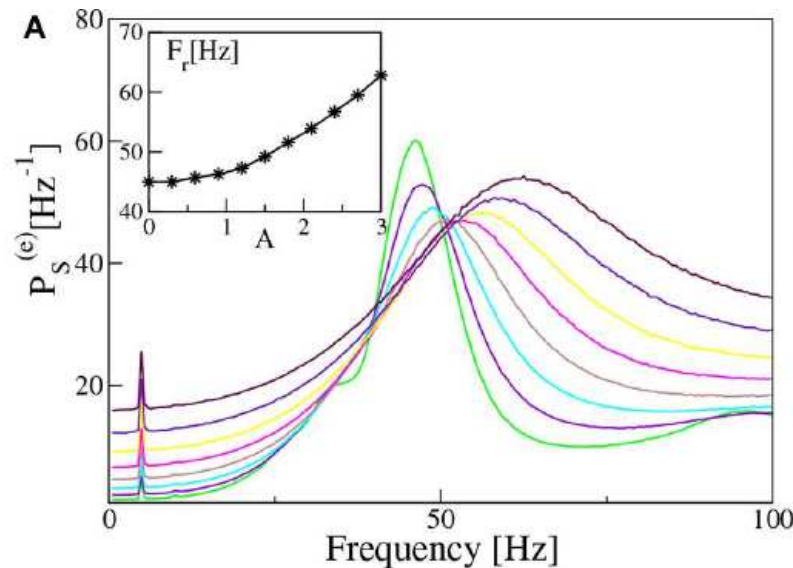
- Black stars : simulations
- Red circles experiments:
  - Filled: Butler et al., J. Neurosci. (2016)
  - Empty: Butler et al., Eur. J. Neuroscience (2018)



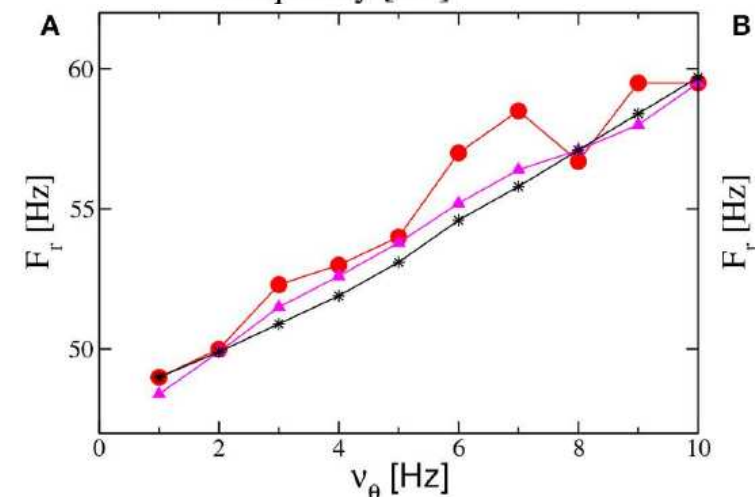


# $\theta$ Nested $\gamma$ Oscillations

## Power Spectra



- $F_r$  grows with  $I_0$  (as in experiments) and with the noise amplitude
- In the experiments  $F_r$  grows with  $\nu_\theta$  : in the simulation this happens only by increasing at the same time the  $\theta$ -power

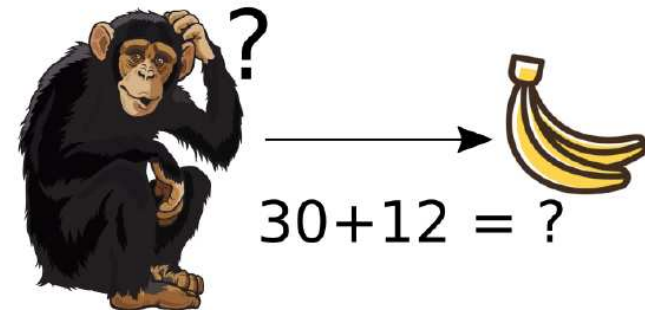


# Synaptic-Based Working Memory

## What is working memory (WM) ?

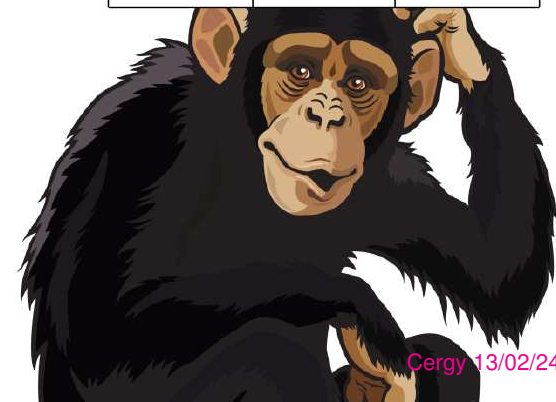
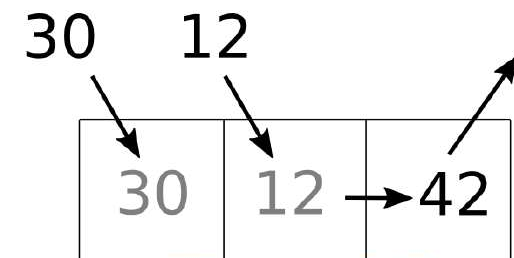
Working memory is a cognitive function fundamental for goal directed behaviours

- solve a task
- achieve the desired goal



Information can be rapidly

- stored & maintained
- processed
- rehearsed

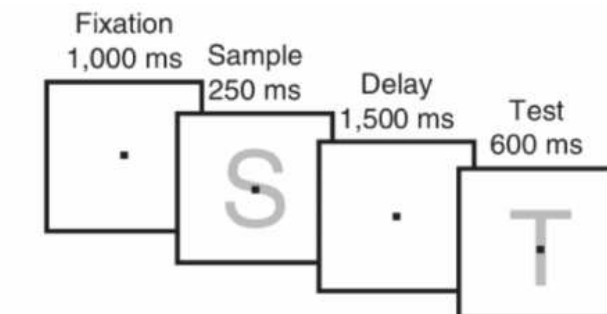
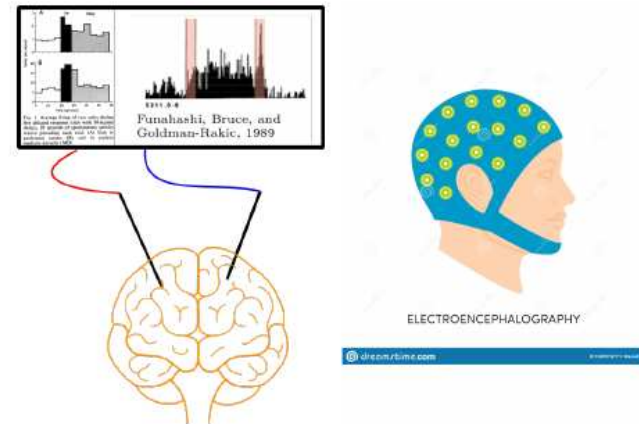


# Synaptic-Based Working Memory

## How to measure WM related activities ?

Train a subject (animal or human) to solve a task requiring short-term storage of information

- Perform electrophysiological measurements:
  - Electrodes: spike trains or local field potentials (LFPs)
  - Electroencephalography (EEG):  
Event Related Potentials (ERPs)
  
- Delayed response paradigm:
  - Present a sample
  - Remove the sample for a delay period
  - Test if new samples match the initial one



Liebe et al., 2012

# Synaptic-Based Working Memory

## Early experimental results

Fuster & Alexander (1971):

- Single unit recording in monkeys in the prefrontal cortex (PFC)
- Sample presentation (cue) evokes increased firing
- Enhanced activity persists in delay period

Information is stored in WM via persistent spiking

Criticisms to the Persistent State Paradigm

- High metabolic cost
- Data processing artifacts (neural spiking averaged over time and across trials)
- Absence of delayed activity in some experiments

Fuster JM, Alexander GE. Neuron activity related to short-term memory. Science. 1971;173(3997):652-654.

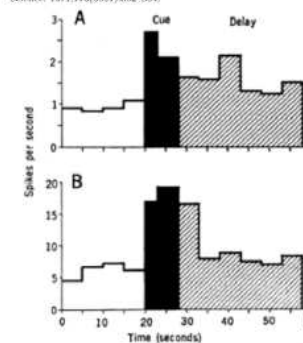
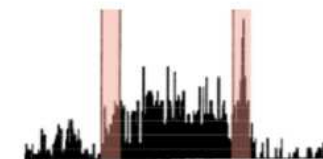
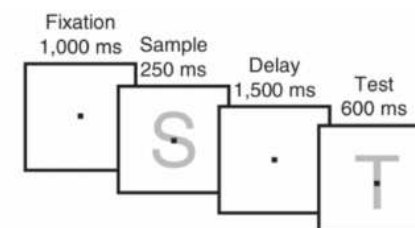


Fig. 1. Average firing of two units during five delayed response trials with 30-second delays, 20 seconds of spontaneous activity record preceding each trial. (A) Unit in prefrontal cortex; (B) unit in nucleus medialis dorsalis (MD).



Funahashi, Bruce, and Goldman-Rakic, 1989



Liebe et al., 2012

# Short-Term Synaptic Plasticity (STP)

A pioneering study revealed that the interactions among pyramidal neurons in the PFC display **synaptic facilitation** lasting hundreds of milliseconds [Wang et al., Nature Neuroscience (2006)] :

**Synaptic Plasticity could be relevant for WM**

Short-term synaptic plasticity can be mimicked with a model developed by Tsodyks, et al. (1998) based on 2 synaptic variable :

**depression**  $x(t)$  and **facilitation**  $u(t)$

The post-synaptic potentials delivered by neuron  $i$  are given in this model by:

$$Jx_i(t)u_i(t)$$

- $x_i(t)$  is the fraction of still **available resources** after neurotransmitter depletion
- $u_i$  is the the fraction of available resources  $x_i$  ready for use :  $u_i x_i$

# Short-Term Synaptic Plasticity (STP)

The evolution in time of these 2 synaptic variables is given by

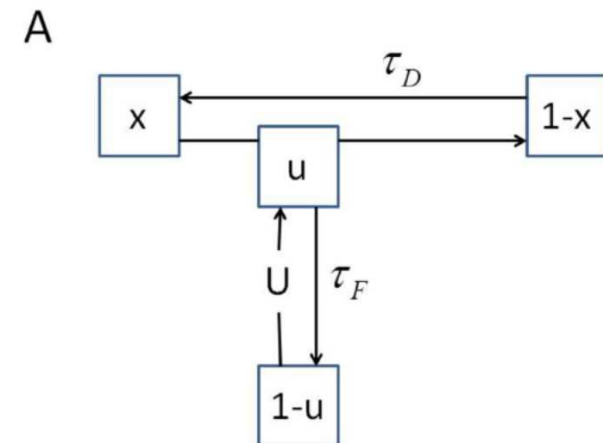
$$\frac{dx_i}{dt} = \frac{1 - x_i}{\tau_d} - u_i x_i \delta(t - t_k^{(i)}) \quad \frac{du_i}{dt} = \frac{U - u_i}{\tau_f} + U(1 - u_i) \delta(t - t_k^{(i)})$$

Following a spike emission,

- $u_i$  increases due to spike-induced calcium influx to the presynaptic terminal
- after which a fraction  $u_i$  of available resources  $x_i$  is consumed to produce the post-synaptic current

Between spikes,

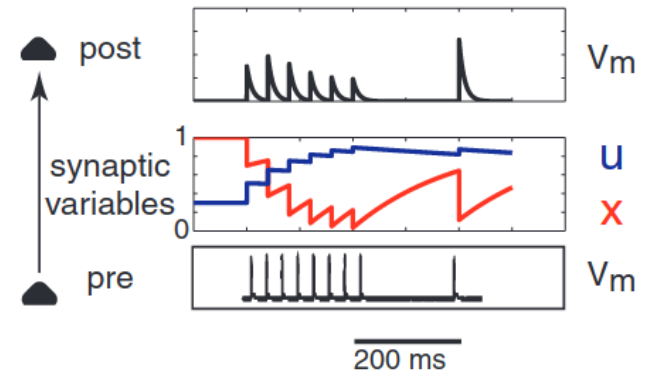
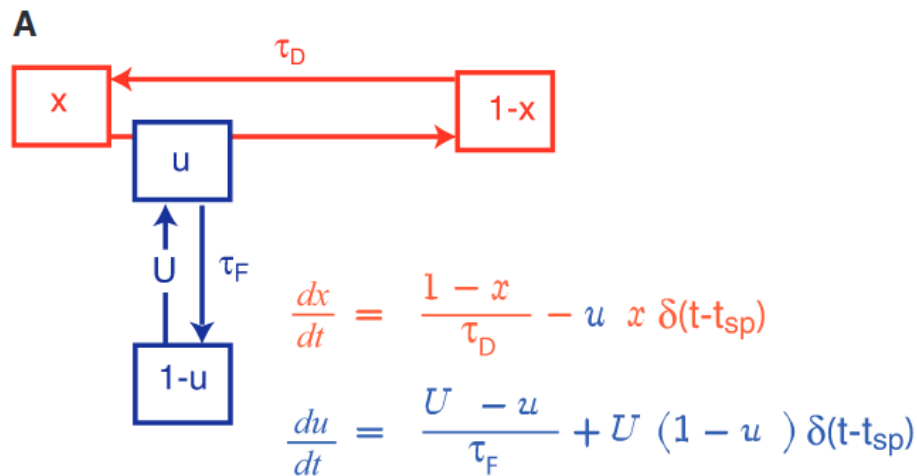
- $u_i$  decays back to  $U$  with time constant  $\tau_f$
- $x_i$  recovers to one with time constant  $\tau_d$



[Tsodyks, Pawelzik, Markram (1998) Neural Computation]

# Short-Term Synaptic Plasticity (STP)

Facilitation Dominated Synapse  $\tau_f = 1500 \text{ ms} \gg \tau_d = 200 \text{ ms}$



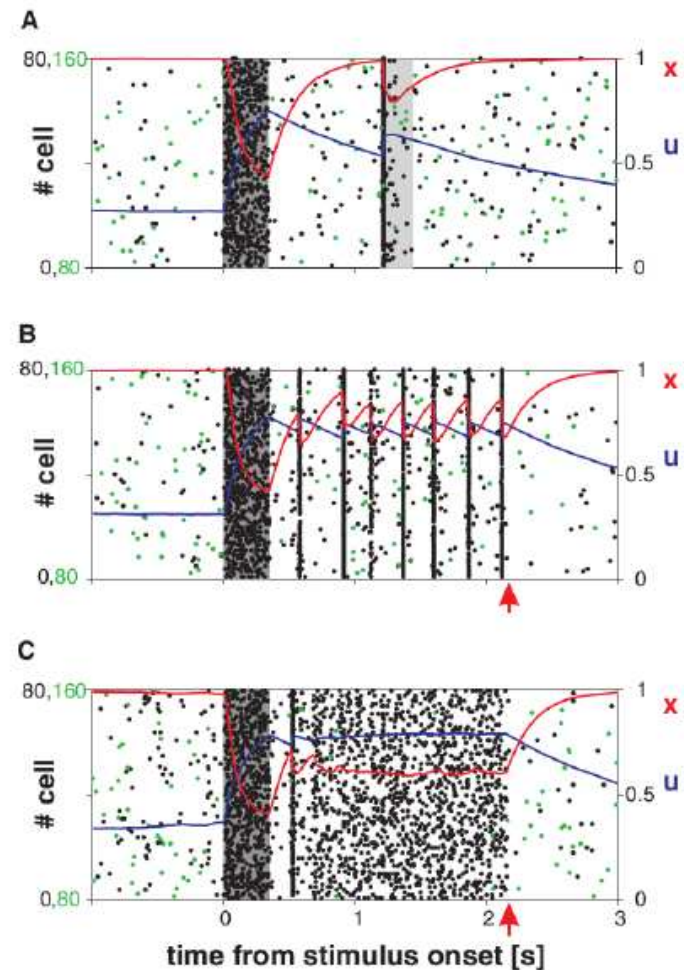
[Tsodyks, Pawelzik, Markram (1998) Neural Computation]

# Synaptic Theory of Working Memory

## A new paradigm

- A spiking network model for WM with synaptic **depression**  $x$  and **facilitation**  $u$
- Cue presentation triggers: **depression**  $x$  & **facilitation**  $u$
- **Depression**: Triggers population bursts needed to refresh memory
- **Facilitation**: Silent WM maintenance & selectivity for unspecific stimuli

[Mongillo, Barak, and Tsodyks, Science (2008)]





# Synaptic Theory of Working Memory

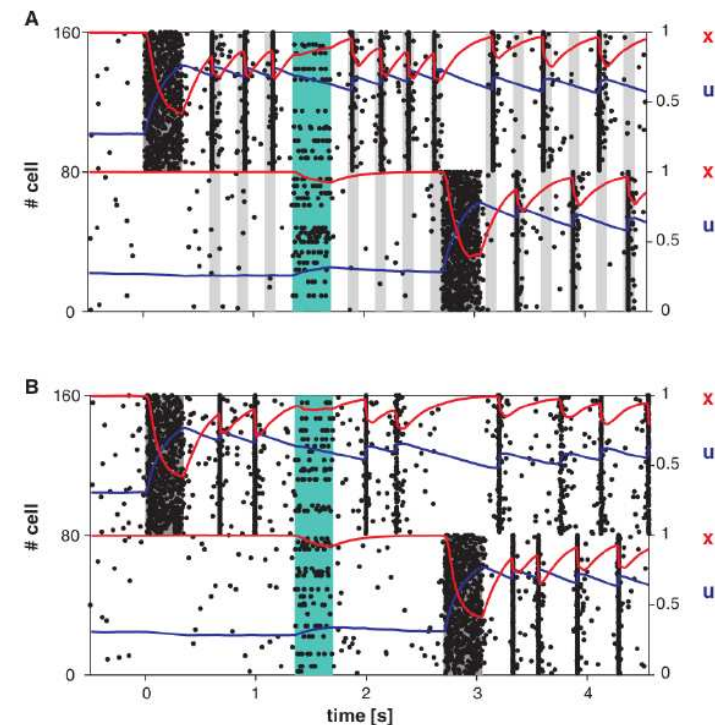
## Benefits of Synaptic Theory of Working Memory

- WM is maintained in **absence of spiking**
- Population bursts allow interference free storage of **more memory items**
- Information is stored at a **population level**

We search for a neural mass model for WM:

- Based on **short-term synaptic plasticity (STP)**
- Able to exhibit **spike synchrony**
- Capable to give acces to **experimental measures (EEGs,LFPs,ERPs)**

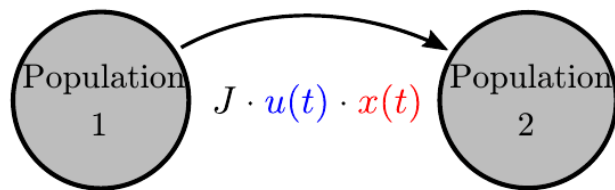
IDEA : extend the MPR model by including STP



# Synaptic-Based Working Memory

## Neural Mass with Synaptic Depression and Facilitation

Short-term synaptic plasticity (STP):



- Depression<sup>4</sup> :  $\dot{x} = \frac{1-x}{\tau_d} - uxr$
- Facilitation<sup>4</sup>:  $\dot{u} = \frac{U-u}{\tau_f} + U(1-u)r$

On population level: macroscopic STP

Neural mass

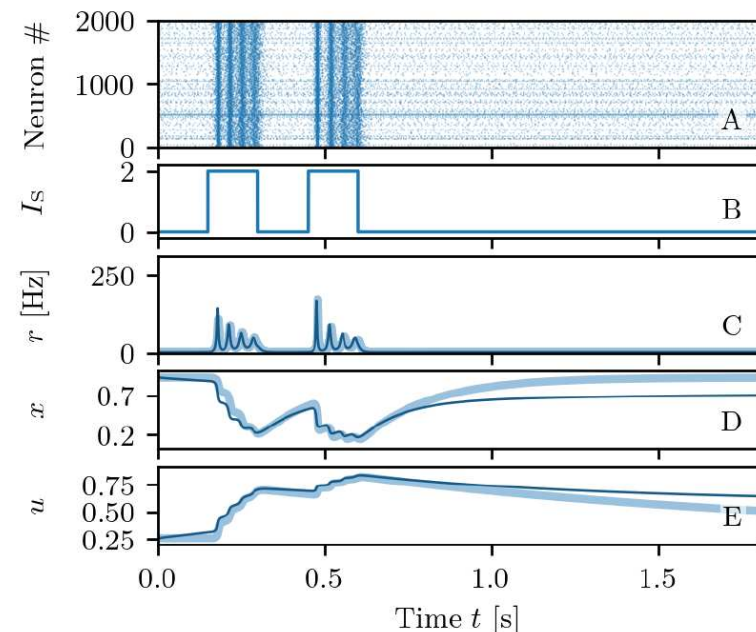
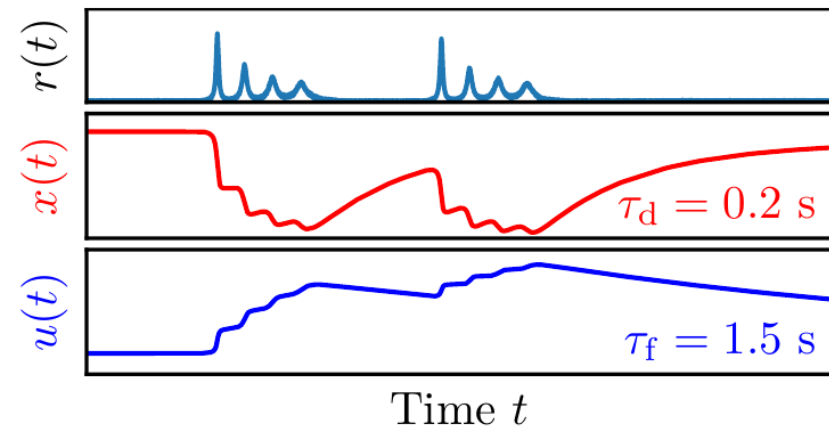
4 equations:

$$\tau_m \dot{r} = \frac{\Delta}{\pi \tau_m} + 2rv$$

$$\tau_m \dot{v} = v^2 - (\pi \tau_m r)^2 + Jux\tau_m r + \bar{\eta}$$

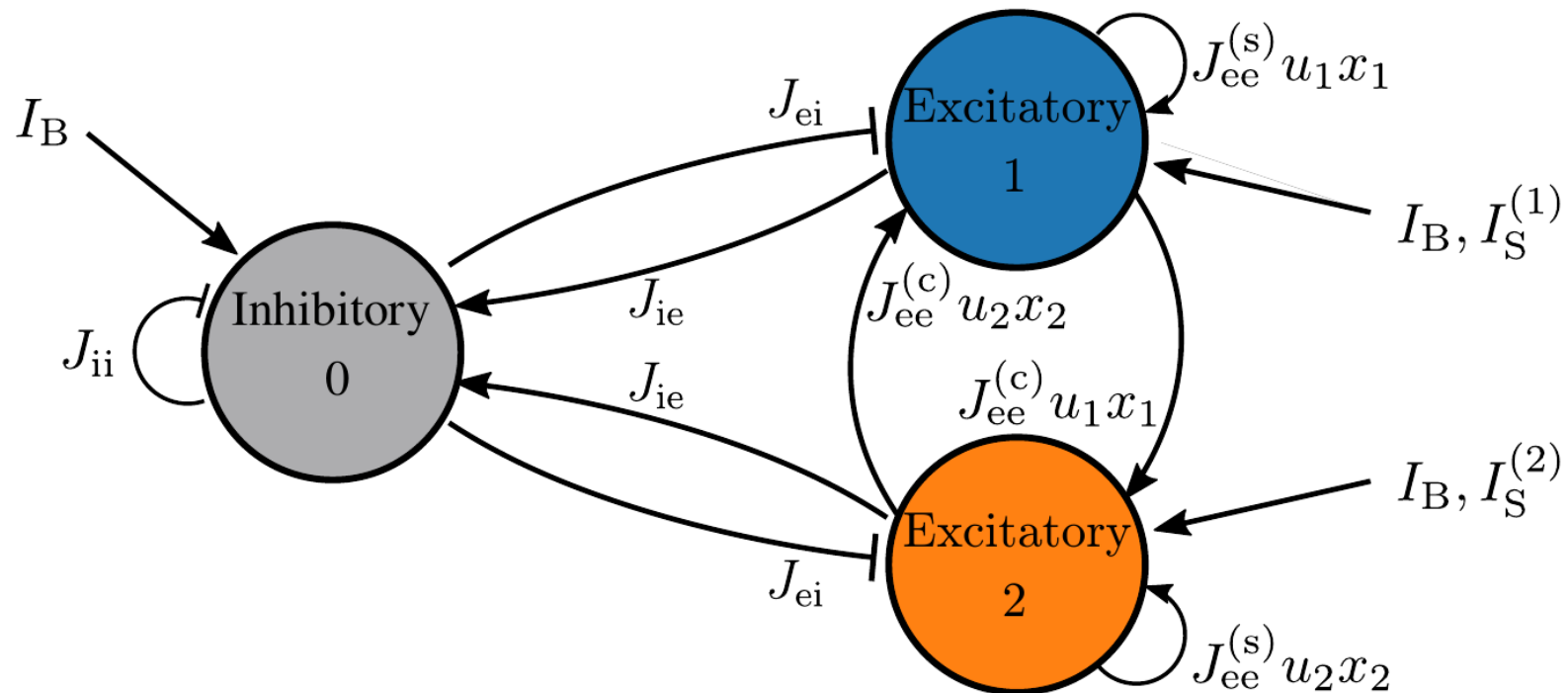
$$\dot{x} = \frac{1-x}{\tau_d} - uxr$$

$$\dot{u} = \frac{U-u}{\tau_f} + U(1-u)r$$



# Synaptic-Based Working Memory

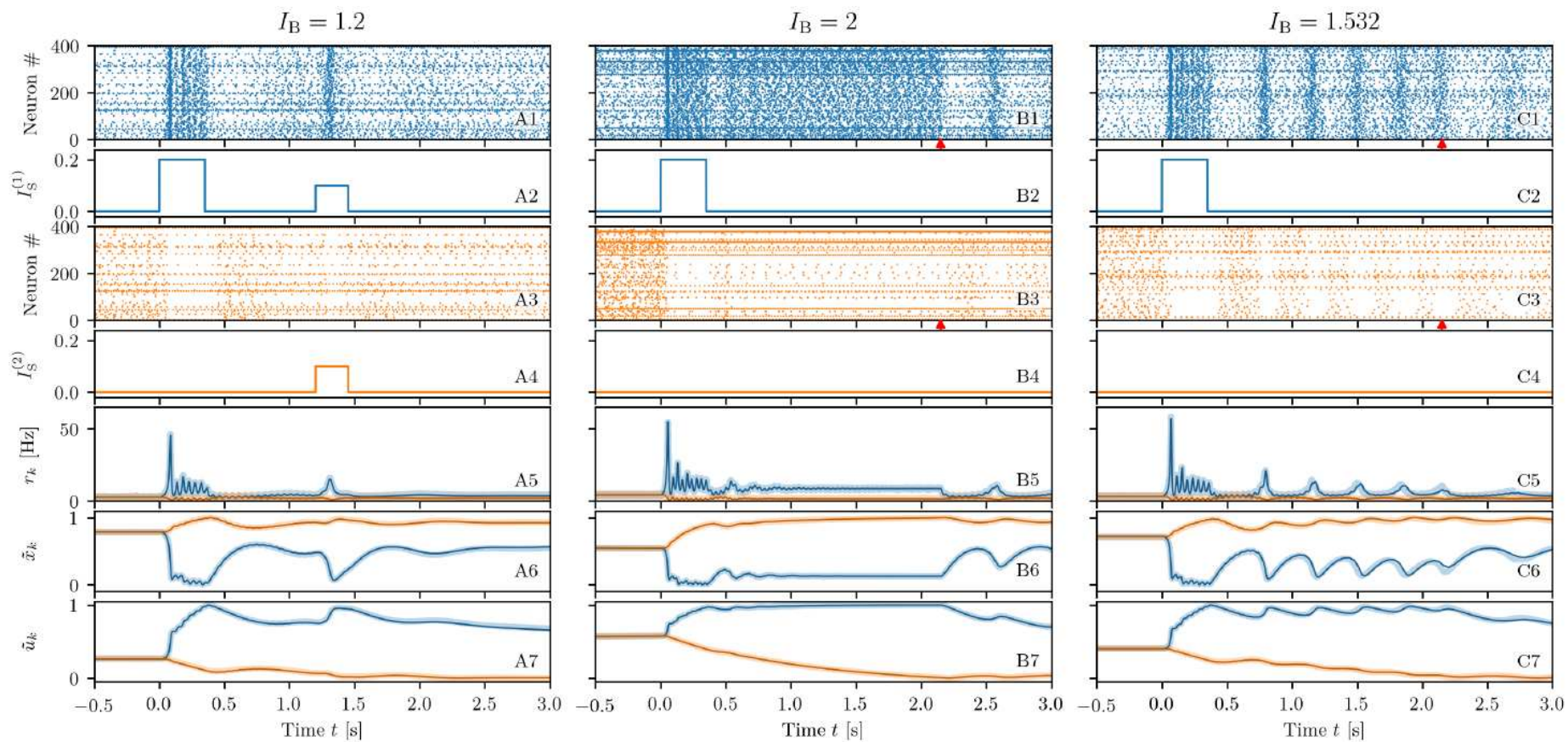
Minimal Architecture to Load up to Two Items in the WM



- Each Item is Loaded in an Excitatory Population
- The Inhibitory Pool avoids Abnormal synchronization
- 3 Neural mass Models ==> 10 Degrees of Freedom
- Comparison with networks with  $N = 600,000$  neurons

# Synaptic-Based Working Memory

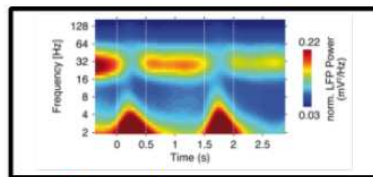
## Loading of One Item in the WM



[Taher, Olmi, AT PLOS Comp Biol (2020)]

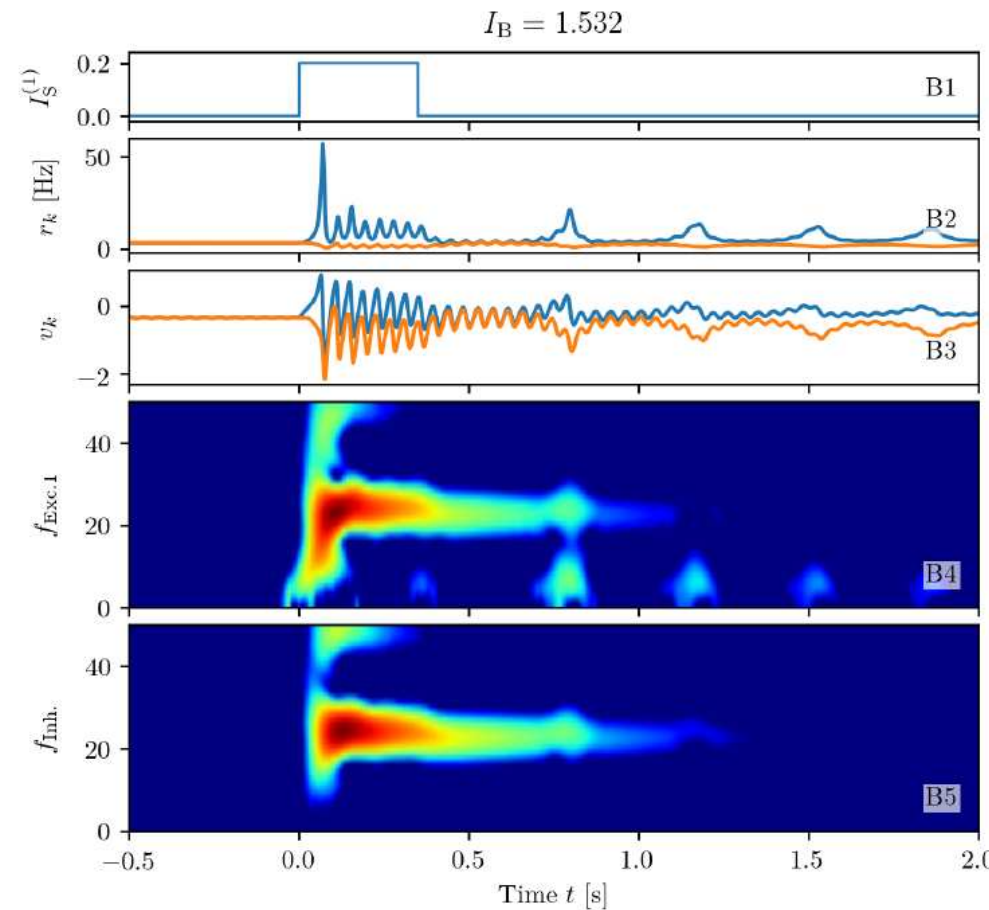
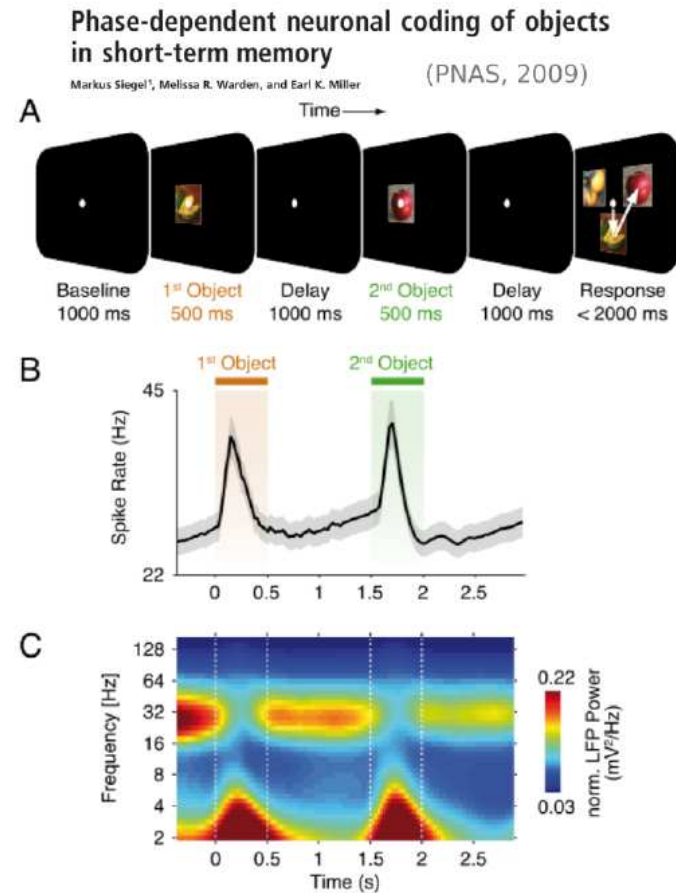
# Comparison with Experimental Data

The novelty of this neural mass is that we have at disposal the mean membrane potential  $V$  and we can estimate the Power Spectra (the Spectrogram) and compare with experimental measurements at a macroscopic scale (LFPs, EEGs, ERPs)



LFP measurements in prefrontal cortex of monkeys during WM tasks

# LFPs in Monkeys



[Taher, Olmi, AT PLOS Comp Biol (2020)]

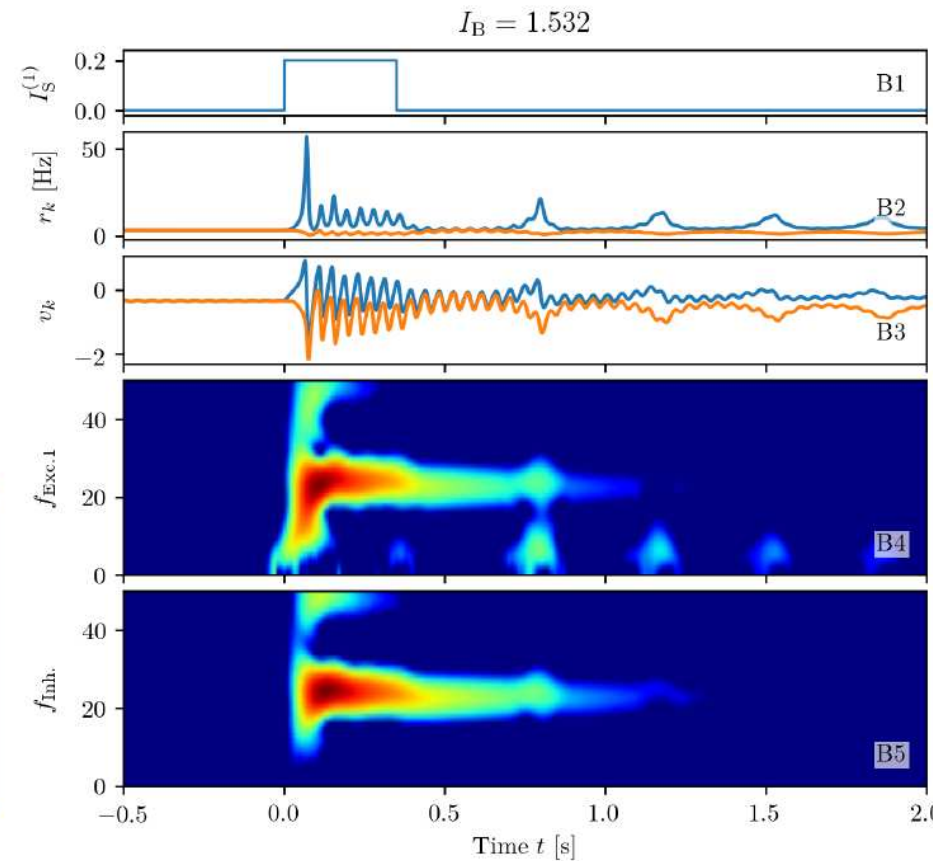
# LFPs in Monkeys

Simulation resembles experimental results:

- $\delta$ -band (2 Hz - 4 Hz) activity locked to stimulus onset
- Sustained  $\beta$ -band (12 Hz - 25 Hz) activity

## Beyond the experiment

- Encoding via PING like mechanism in  $\beta$ -band
- Excitatory population bursts generate  $\delta$ -band signal

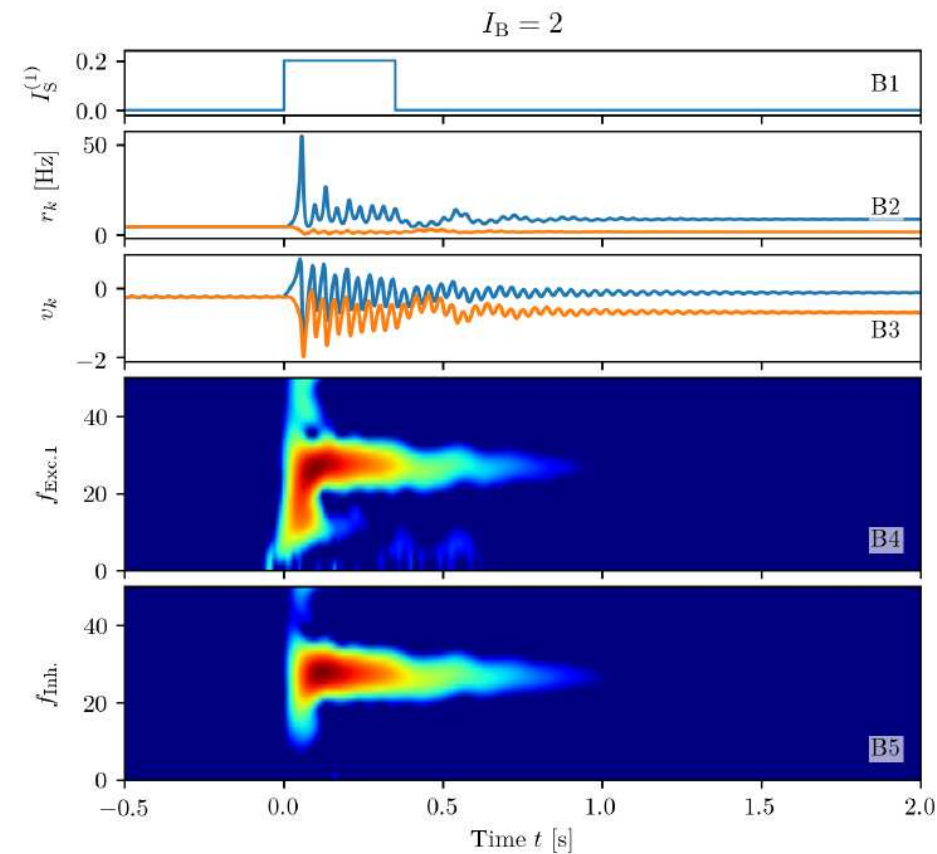
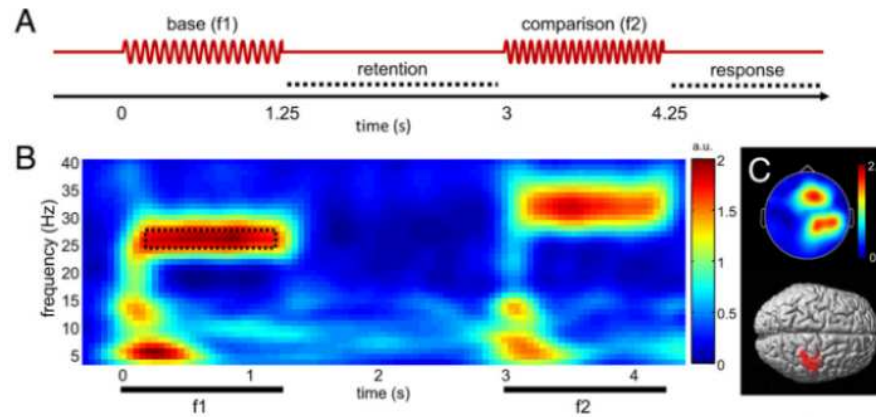


[Taher, Olmi, AT PLOS Comp Biol (2020)]

# EEGs in Humans

## Oscillatory Correlates of Vibrotactile Frequency Processing in Human Working Memory (The Journal of Neuroscience, 2010)

Bernhard Spitzer, Evelin Wacker, and Felix Blankenburg  
Department of Neurology and Bernstein Center for Computational Neuroscience, Charité, 10115 Berlin, Germany



[Taher, Olmi, AT PLOS Comp Biol (2020)]

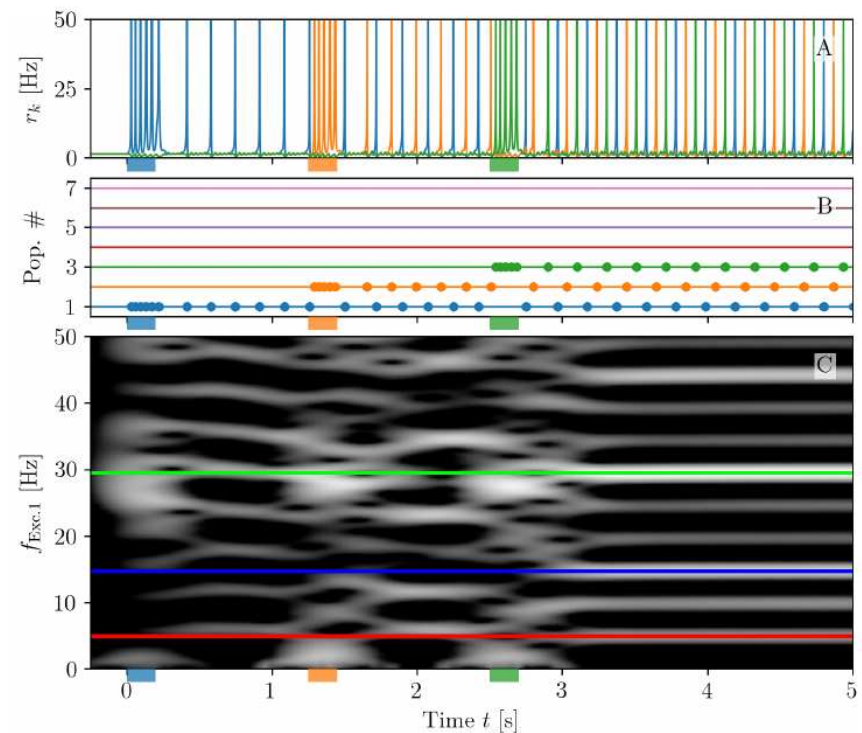


# Multi-item Loading in WM

Multiple items can be loaded in an architecture with 7 excitatory populations and 1 inhibitory pool

## Three Item Loading in WM

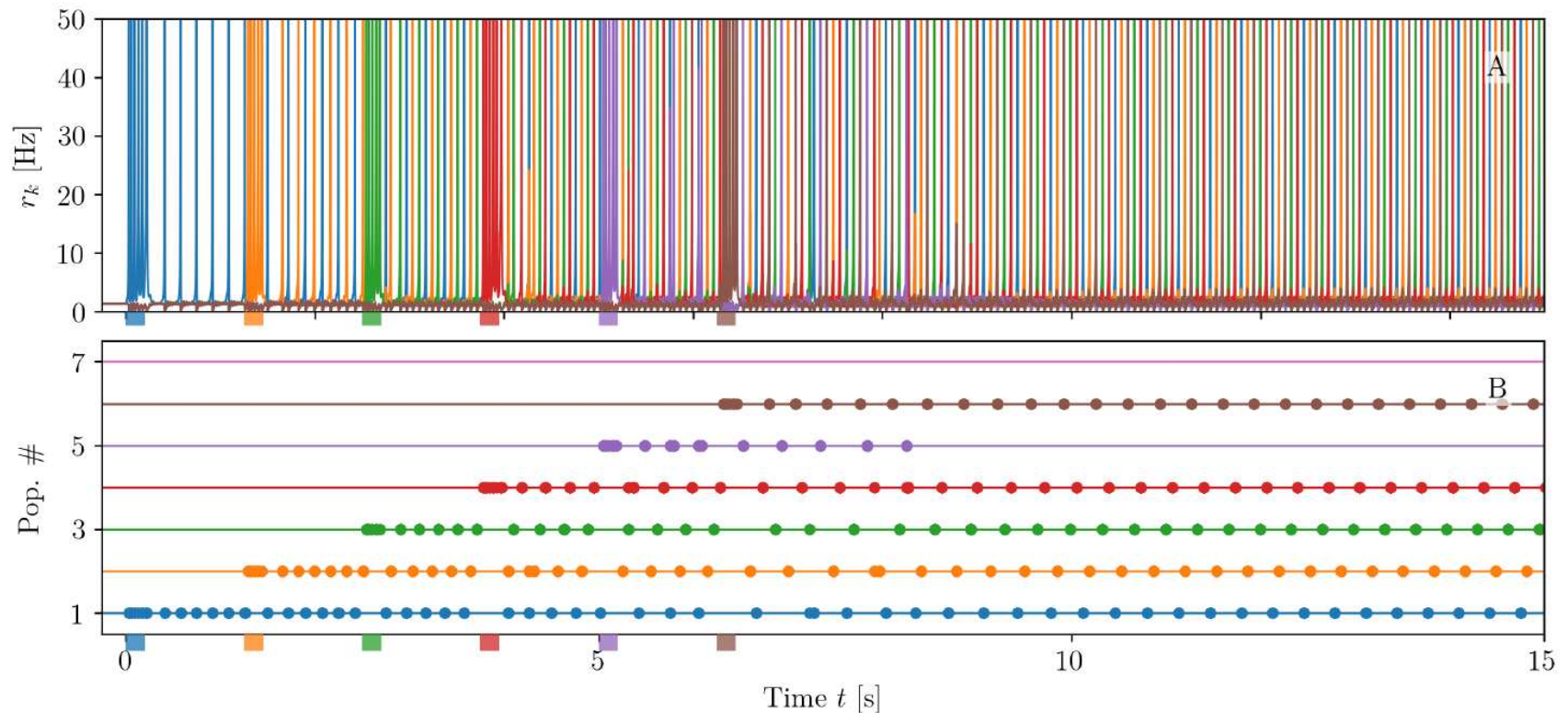
- Activity in the  $\delta$ -band upon item presentation
- Fundamental memory cycle frequency  $f_{cycle}$
- Frequency of burst emission  $f_{burst}$
- Resonance with the  $\beta$ -band sub-threshold oscillations



[Taher, Olmi, AT PLOS Comp Biol (2020)]

# Working Memory Capacity

A series of studies have investigated the working memory capacity and indicated as maximal number of stored items  $N_c \simeq 3 - 5$

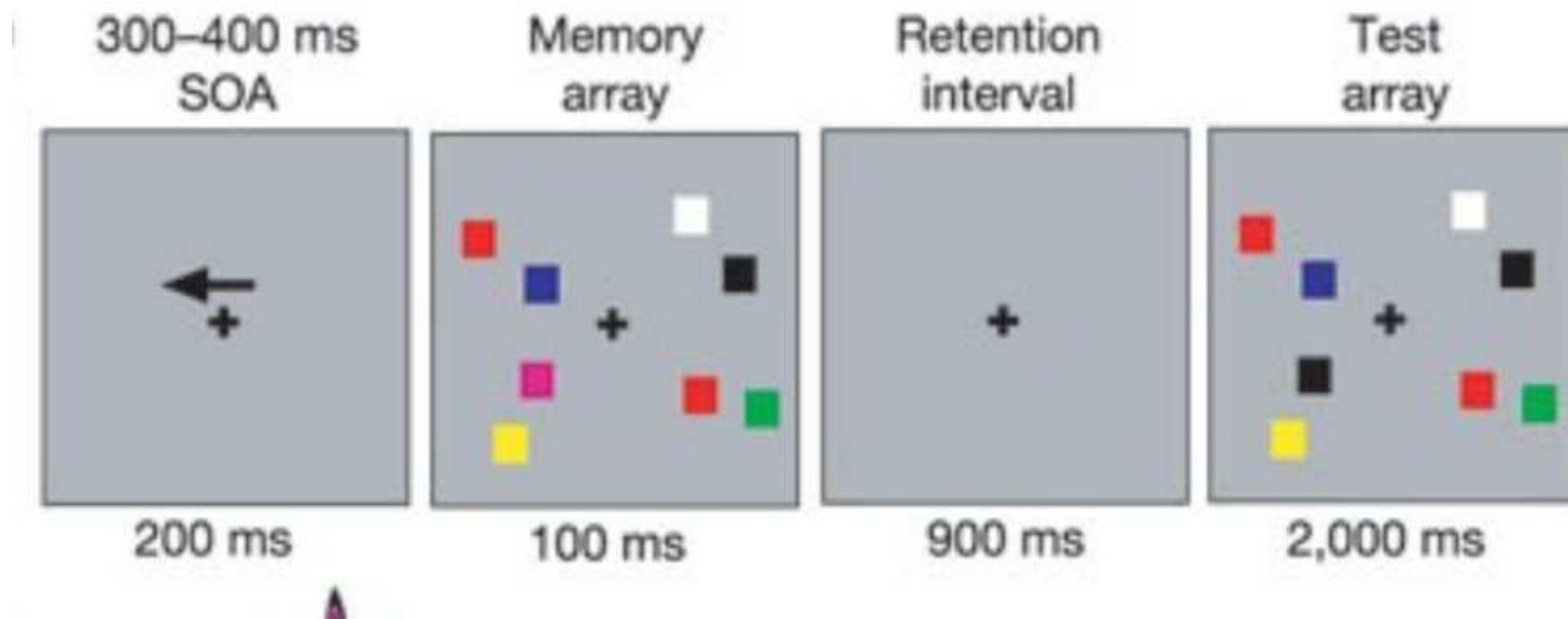


[Cowan N, Behavioral and brain sciences (2001); Cowan N, Current directions in psychological science (2010)]

# Working Memory Capacity

How to measure the WM Capacity from neurophysiological data ?

Vogel et al. (Nature 2004, Nature 2005) introduced a measure of the WM capacity on humans based on **event-related potentials (ERPs)** from adults performing a visual memory task.



The task consists in memorizing an array of  $N_L$  colored squares

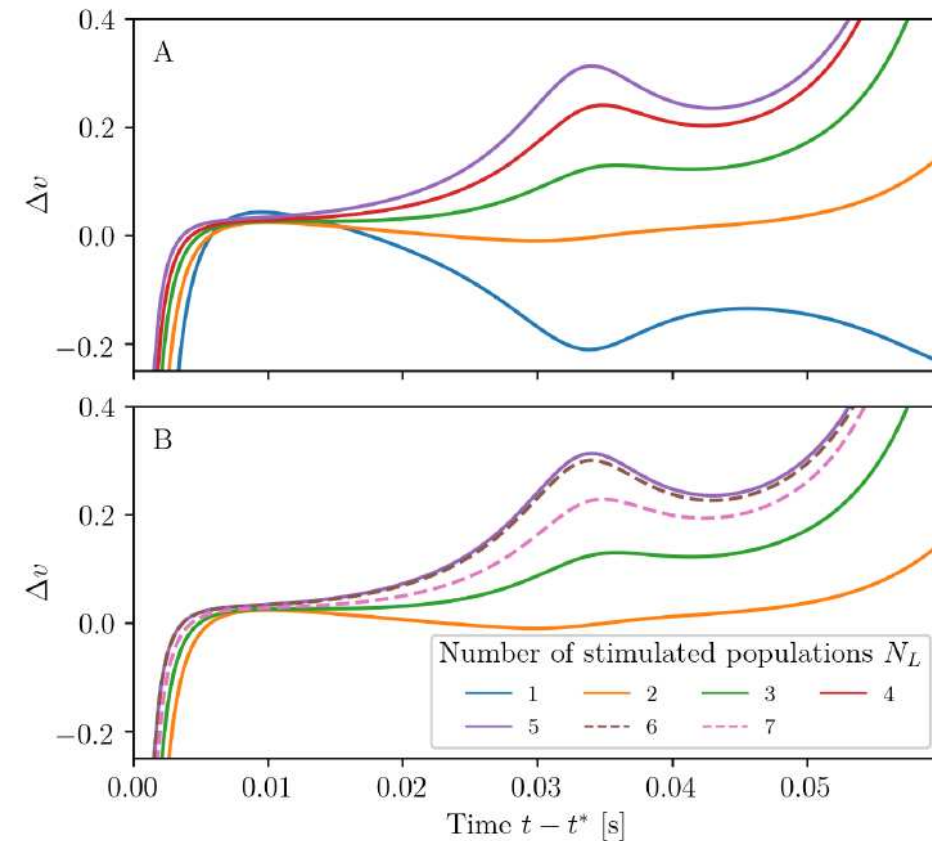
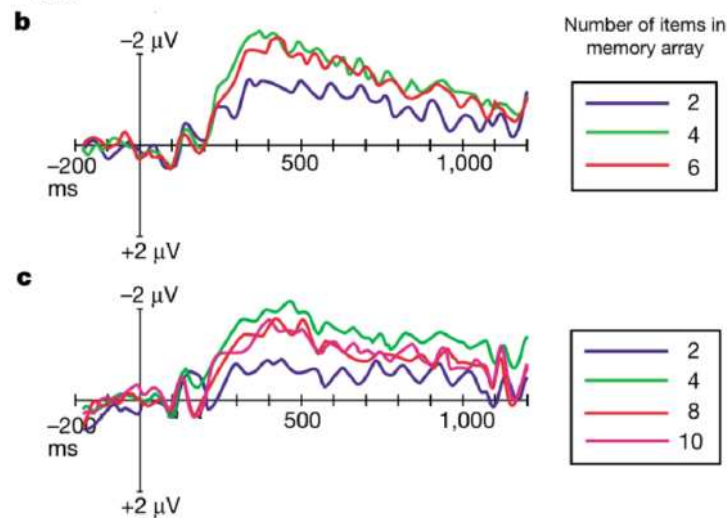
# Working Memory Capacity

The mean membrane potential can be employed, analogously to the ERP in the experiments, as a proxy to measure the memory load and capacity

## Neural activity predicts individual differences in visual working memory capacity (Nature, 2004)

Edward K. Vogel & Maro G. Machizawa

Department of Psychology, University of Oregon, Eugene, Oregon 97403-1227, USA



# Neural Mass for Fluctuation Driven Populations

- The MPR Neural Mass (PRX,2015) reproduces the dynamics of one **heterogeneous** population of **globally coupled neurons**
- The MPR model can be extended to two or more interacting populations
- The MPR model can be extended to the whole connectome:  
[V. Jirsa, S. Olmi, G. Rabuffo, et al, bioRxiv 2 preprints (2021)]
- The MPR model has been extended to encompass **delay, gap junctions, short-term plasticity, asymmetric spike forms, conductance based neurons etc**

However so far this Neural Mass always concerns **globally coupled neurons** without **noise sources**, a non realistic representations of neural systems, which always present:

- background noise
- random distribution of the synaptic connections

May we develop a Neural Mass encompassing quenched and dynamical disorder sources ?

YES WE DID !

# Neural Mass for Fluctuation Driven Populations

- The MPR model is based on the assumption that the distribution of the membrane potentials is Lorentzian (LD),
- The presence of dynamical disorder modifies the LD, which is now distorted
- The LD cannot be expanded in regular cumulants or moments, they all diverge

We have introduced an expansion of the LD in pseudo-cumulants, to treat distorted LD

This allow to derive a neural mass encompassing different sources of noise

- A low dimensional mean-field model reproducing the dynamics of spiking QIF neurons
  - subject to background noise
  - and/or arranged in sparse random network

[Goldobin, diVolo, AT, "A reduction methodology for fluctuation driven population dynamics", PRL (2021)]

# The Model

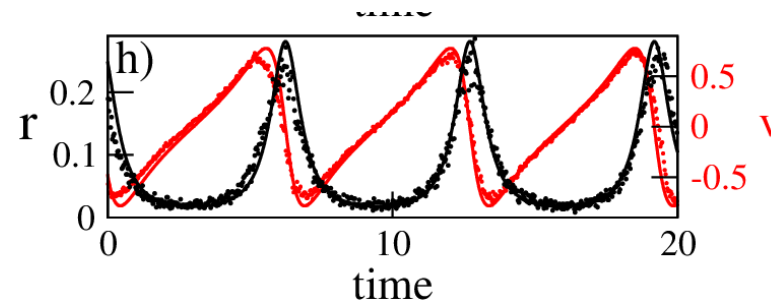
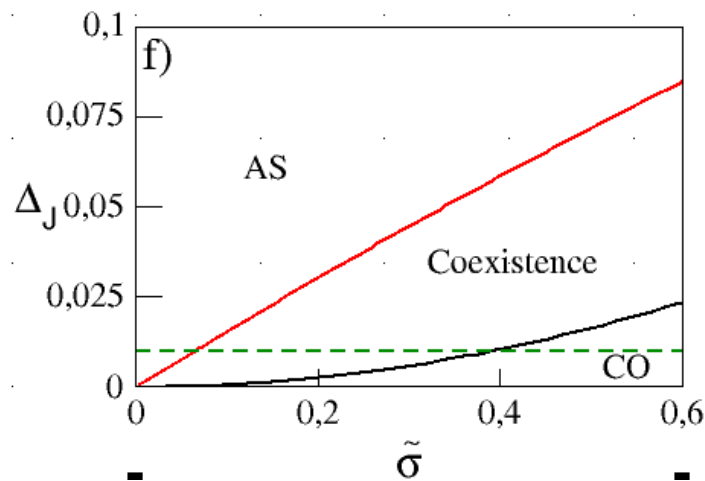
For a heterogeneous population of QIF neurons subject to different noise sources we can derive a 4 dimensional neural mass model

$$\begin{aligned}\dot{r} &= \frac{\Delta_{\eta} + \Delta_J r + p_2}{\pi} + 2rv \\ \dot{v} &= I_0 + \eta_0 + J_0 r - \pi^2 r^2 + v^2 + q_2 \\ \dot{q}_2 &= 2\mathcal{N}_R + 4(q_2 v - \pi p_2 r) \\ \dot{p}_2 &= 2\mathcal{N}_I + 4(\pi q_2 r + p_2 v)\end{aligned}$$

- Four Macroscopic Variables :
  - Mean Membrane Potential and Firing Rate  $(v, r)$
  - Two Dynamical Variables for the LD Distortions  $(q_2, p_2)$
- Quenched heterogeneities
  - Excitabilities : LD with  $(\eta_0, \Delta_{\eta})$
  - Synaptic Couplings : LD with  $(J_0, \Delta_J)$
- Noise Sources are encompassed in the terms  $(\mathcal{N}_R, \mathcal{N}_I)$

# Background Noise

Globally coupled inhibitory network of QIF neurons each subject to independent additive Gaussian noise of variance  $\sigma^2$  :  $(\mathcal{N}_R = \sigma^2, \mathcal{N}_I = 0)$

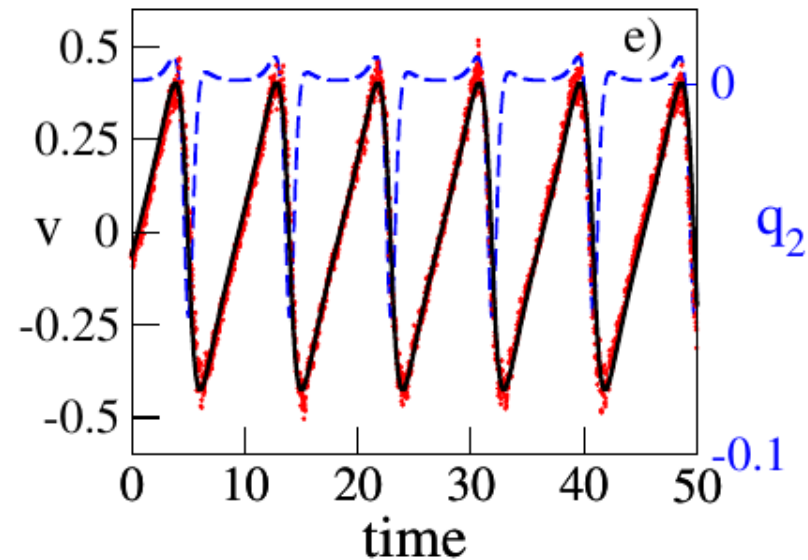
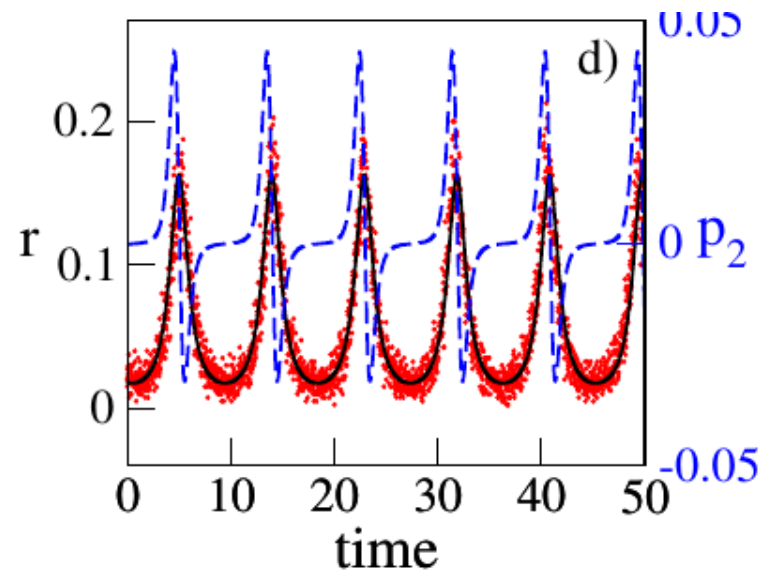


- the bifurcation diagram reveals Asynchronous States (ASs), Collective Oscillations (COs) and regions of coexistence of ASs and COs
- the neural mass results are in agreement with the network simulations
- the MPR model cannot capture even qualitatively these regimes displayed by the noisy spiking network
- Noise induced COs in absence of any synaptic or delay time scale



# Sparse Networks

- Random network with in-degrees  $k_j$  distributed as a LD, with median  $K$  and HWHM  $\Delta_K = \Delta_0 K$ .
- We assume that each neuron receive  $k_j$  independent Poissonian spike trains with rate  $r$
- This amounts to set  $(\mathcal{N}_R = (J_0^2 r)/(2K), \mathcal{N}_I = -\Delta_0 \mathcal{N}_R)$



# Summary

The **Next Generation Neural Masses** of MPR open a complete new perspective for realistic simulations of heterogenous spiking networks

- The Neural Masses reproduce with high fidelity the network dynamics:
  - not only the firing rate but also the sub-threshold membrane potential dynamics
  - synchronization and de-synchronization phenomena
- The Neural Masses reproduces relevant neuroscience phenomena:
  - $\theta$ - $\gamma$  cross coupling in the hippocampus and other areas
  - fast and slow  $\gamma$  oscillations in the hippocampus
  - working memory processes
  - spike frequency adaptation
- the neural mass MPR can be extended to capture fluctuation driven phenomena present in realistic brain circuits

[Ott, & Antonsen Chaos (2008)] 1209 citations

[Montbrió, Pazó, Roxin, Physical Review X, 2015] 405 citation

# Collaborators



Simona Olmi



Halgurd Taher



David Angulo



Andrea Ceni



Marco Segneri



Honjie Bi



Matteo di Volo



Denis Goldobin

# Publication List

- M. di Volo, AT, "Transition from asynchronous to oscillatory dynamics in balanced spiking networks with instantaneous synapses", Phys. Rev. Lett. (2018)
- H. Bi, M. Segneri, M. di Volo, AT "Coexistence of fast and slow gamma oscillations in one population of inhibitory spiking neurons", Physical Review Research (2020)
- A.Ceni, S. Olmi, AT, D. Angulo Garcia, "Cross frequency coupling in next generation inhibitory neural mass models", Chaos (2020)
- M. Segneri, H.Bi, S. Olmi, AT, "Theta-nested gamma oscillations in next generation neural mass models", Frontiers in Computational Neuroscience (2020)
- H. Taher, AT, S. Olmi, "Exact neural mass model for synaptic-based working memory", PLOS Computational Biology (2020)
- D. Goldobin, M diVolo, AT, "A reduction methodology for fluctuation driven population dynamics", Phys Rev Lett (2021)
- A. Ferrara, D. Angulo-Garcia, AT, S. Olmi, "Population spiking and bursting in next generation neural masses with spike-frequency adaptation", Physical Review E (2023)
- Y. Feld, A. K. Hartmann, AT, "Coexistence of asynchronous and clustered dynamics in noisy inhibitory neural networks", New Journal of Physics (2024)