

Next generation neural mass models

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Plan of the Talk



 The Quadratic Integrate and Fire (QIF) Neuron
 Derivation of the Neural Mass Model (MPR) (Montbrió, Pazó, Roxin, PRX, 2015)
 Applications of the Neural Mass Model to Interacting Populations (2020)
 Cross frequency coupling
 Theta-nested gamma oscillations
 Synaptic-based working memory

Extension of the neural mass to fluctuation driven population dynamics (2021)

- Neural Networks with Background Noise
- Neural Networks with Random Sparse Synaptic Couplings



Quadratic Integrate and Fire Neuron

This a very simple model, which can mimick realistic neural dynamics

$$\frac{dV}{dt} = V^2 + \eta$$

 \blacksquare V membrane potential with threshold V_{th} and reset V_r

 η neural excitability

 $\blacksquare \eta > 0$ Tonic Neuron

 $\eta \leq 0$ Excitable Neuron

The model has been developed to reproduce parabolic bursting in neurons of the Aplysia abdominal ganglion (when sinusoidally forced) and low firing neurons (Class I). Ermentrout and Kopell, (1986) SIAM Journal on Applied Mathematics Latham et al. (2000) Journal of Neurophysiology



Quadratic Integrate and Fire Neuron



Tonic neuron $\eta > 0$

The firing period is $T_{firing} = \frac{\pi}{\sqrt{\eta}}$

Excitable neuron $\eta \leq 0$

- If $V(t_0) \le \sqrt{|\eta|}$ Subthreshold Dynamics $V \to V_{rest} = -\sqrt{|\eta|}$
- If $V(t_0) > \sqrt{|\eta|}$ emission of a spike followed by relaxation to V_{rest}



Population of QIF neurons



The evolution of the membrane potentials of N globally coupled heterogeneous neurons can be written as

$$\frac{dV_j}{dt} = V_j^2 + \eta_j + I_e(t) + Jr(t)$$

where

- η_j is the excitability of neuron j
- $I_e(t)$ is some external current
 - J is the synaptic coupling -J > 0 (J < 0) excitatory (inhibitory) neurons

$$r(t) = \frac{1}{N} \sum_{j} \sum_{k(j)} \delta(t - t_{k(j)}) = \frac{1}{N} \sum_{j} r_j(t)$$

is the average firing rate due to all the post-synaptic potentials (δ -spikes) emitted by all neurons in the network



In the limit $N \to \infty$ we can describe the population of N neurons in term of a probability density function (PDF) $\rho(V|\eta, t)$ of their membrane potentials

Continuous formulation

 $\rho(V|\eta, t)dV$ = fraction of neurons with membrane potentials between V and V + dV, with excitability η , at time t

The excitabilities $\eta_j \Rightarrow$ are continuous random variable distributed according to a PDF $g(\eta)$

Since the number of neurons should be conserved in time, the PDF ρ satisfies the following

Continuity equation $\partial_t \rho + \partial_V [(V^2 + \eta + Jr + I_e)\rho] = 0$

The stationary solution ρ_0 (for constant I_e) is given by

$$\rho_0(V|\eta) \propto \frac{1}{V^2 + \eta + Jr + I_e}$$

this is a Lorentzian (or Cauchy) Distribution $L(x) = \frac{\Delta}{\pi(x^2 + \Delta^2)}$



Lorentzian Ansatz

MPR assumed that also for non-stationary case the distribution is Lorentzian at any time

$$\rho(V|\eta, t) = \frac{1}{\pi} \frac{x(\eta, t)}{[V - y(\eta, t)]^2 + x(\eta, t)^2}$$

where the median $y(\eta,t)$ and the HWHM $x(\eta,t)$ correspond to the

Mean Membrane Potential

Firing Rate

 $v(\eta,t)=y(\eta,t)$

for the neurons with excitability η

$$r(\eta, t) = \frac{x(\eta, t)}{\pi}$$

for the neurons with excitability η



Therefore within the Lorentzian Ansatz the continuity equation can be rewritten as follows for the complex variable $w(\eta, t) = \pi r(\eta, t) + iv(\eta, t)$

$$\partial_t w(\eta, t) = i[-w^2(\eta, t) + \eta + Js + I]$$

This is a neural mass equation describing the dynamics of a population of neurons with the same excitability η in terms of their firing rate $r(\eta, t)$ and their mean membrane potential $v(\eta, t)$.

However for a heterogeneous population one has still an extremely large number of equations corresponding to all the possible values of the excitability η_i

How can we reduce the system to a low dimensional one ?



Systems with random heterogeneities have been treated exactly in statistical mechanics by assuming a Lorentzian distribution for the heterogeneities E. Yakubovich, Soviet Physics JETP. 1969

$$g(\eta) = rac{1}{\pi} rac{\Delta}{(\eta - ar{\eta})^2 + \Delta^2}$$

This allows to estimate exactly the average mean membrane potential and firing rate

$$v(t) = \int_{-\infty}^{+\infty} v(\eta, t) g(\eta) d\eta \qquad r(t) = \int_{-\infty}^{+\infty} v(\eta, t) g(\eta) d\eta$$

2-dimensional Neural Mass Model

$$\dot{r} = \frac{\Delta}{\pi} + 2rv$$
 $\dot{v} = v^2 + \bar{\eta} + Jr + I(t) - \pi^2 r^2$

[Montbrió, Pazó, Roxin, Phys. Rev. X (2015)]



MPR Model



2-dimensional Neural Mass Model

$$\dot{r} = \frac{\Delta}{\pi} + 2rv \qquad \dot{v} = v^2 + \bar{\eta} + Jr + I(t) - \pi^2 r^2$$

This Neural Mass Model describes exactly the dynamics of the Network done of $N~{\rm QIF}$ neurons

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[Montbrió, Pazó, Roxin, Phys. Rev. X (2015)]
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Fast γ oscillations emerge in recurrently coupled inhibitory networks

- An external excitatory drive tends to synchronize the inter-neuronst that fire together
- The firing of the inhibitory neurons leads to their silencing on a timescale dictated by the duration τ_d of the post-synaptic potentials *S*
- This mechanism generates collective oscillations in the network [Whittington, Traub, Jefferys, Nature. 1995]



If $\tau_d >> \tau_m$, the synaptic time scale is longer that that the membrane time scale no oscillations, the recurrent input appears as an average inhibitory current [DeValle, Roxin, Montbrió, PLOS Comp Biol (2017)]





Ceni, Olmi, AT, Angulo Garcia, Chaos (2020)





For increasing heterogeneity ($\Delta = 0.01$, $\Delta = 0.05$, $\Delta = 0.1$, $\Delta = 0.14$) the observation of collective oscillations requires finer tuning of the parameters

Frequency decreases for increasing au_d and J



The heuristic firing rate models (e.g. Wilson-Cowan model)

$$\tau_m \dot{r} = -r + F(-\tau_m J^{(ii)}S + I^{(i)})$$

$$\tau_d \dot{S} = -S + r$$

do not display Collective Oscillations without introducing an effective delay which takes somehow in account for the sub-threshold dynamics

- Firing rate models do not encompass Subthreshold Dynamics of the Membrane potential
- The membrane potential dynamics is fundamental to have synchronization effects
- The MPR model reproduces network dynamics in this case, Wilson-Cowan not

[DeValle, Roxin, Montbrió, PLOS Comp Biol (2017)]



One of the most present cross-frequency coupling in the brain is the interaction between slow θ -rhytms (5-10 Hz) and fast γ -oscillations (20-100 Hz)

Which is the origin of this CFC ?

White, Banks, Pearce, and Kopell, PNAS 2000 proposed that θ - γ CFC emerges due to the interaction of two inhibitory populations with different kinetic properties



May we observe this phenomenon for two coupled neural masses *A* and *B* with exponential synapses with different kinetics ?

- A slow inhibitory population A with self-coupling $J_{AA} = -2$,
- A fast inhibitory population B with self-coupling $J_{BB} = -11$
- Cross-couplings J_{AB} (A \rightarrow B) and J_{BA} (B \rightarrow A)



Fast population: $\tau_{A,d} = 9 \text{ ms}$ - Slow population: $\tau_{B,d} = 50 \text{ ms}$

3 : 1 phase-locking mode: $\theta - \gamma$ coupling



 \Rightarrow For large values of J_{AB} and disorder Δ , the 3:1 locked mode is lost

[Ceni, Olmi, AT, Angulo Garcia, Chaos (2020)]



External θ -forcing on the slow population $\theta \simeq 10$ Hz

Fast population: $\tau_{A,d} = 9 \text{ ms}$ - Slow population: $\tau_{B,d} = 50 \text{ ms}$



 \Rightarrow Power in the θ band is increased

 \Rightarrow Adding the (slow) modulation to the slow population increases the amount of disorder (Δ) that can sustain the $\theta - \gamma$ coupling

[Ceni, Olmi, AT, Angulo Garcia, Chaos (2020)]



 $I_B = I_0 \sin(2\pi 10t)$



 $\Delta = 0.05$

Phase-phase CFC:

collective oscillations of the slow population an of the fast one are locked in phase

 $\Delta = 0.2$

Phase-amplitude CFC: the slow population modulates the amplitude of the oscillations of the fast one



Several optogenetic experiments performed in different areas of the hippocampus and entorhinal cortex suggest that a θ frequency drive is sufficient to induce in vitro θ - γ CFCs



- Butler et al. (2016) stimulated CA1 pyramidal neurons with an excitatory drive $I_{\theta} = I_0 \sin(\nu_{\theta} t)$ with $\nu_{\theta} \in [1:10]$ Hz in vitro
- They show that this is sufficient to generate intrinsic CA1 γ oscillations in vitro similar properties to in vivo CA1 γ oscillations.
- They suggest that the mechanism for the generation of the oscillations is of the pyramidal interneuron gamma (PING) type





Excitatory-Inhibitory Neural Masses

- We consider two coupled populations : an excitatory and an inhibitory plus an external periodic forcing on the excitatory population
- We observe θ nested γ oscillations as in the experiments



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- F_r : response frequency
- I_0 : forcing amplitude
- P_{γ} : power under the main γ -peak







[Segneri, Bi, Olmi, AT Front.



What is working memory (WM) ?

Working memory is a cognitive function fundamental for goal directed behaviours

solve a task

achieve the desired goal

Information can be rapidly

- stored & maintained
- processed
- rehearsed







How to measure WM related activities ?

Train a subject (animal or human) to solve a task requiring short-term storage of information

- Perform electrophysiological measurements:
 - Electrodes: spike trains or local field potentials (LFPs)
 - Electroencephalography (EEG): Event Related Potentials (ERPs)
- Delayed response paradigm:
 - Present a sample
 - Remove the sample for a delay period
 - Test if new samples match the initial one





Early experimental results

Fuster & Alexander (1971):

- Single unit recording in monkeys in the prefrontal cortex (PFC)
- Sample presentation (cue) evokes increased firing
- Enhanced activity persists in delay period

Information is stored in WM via persistent spiking

Criticisms to the Persistent State Paradigm

- High metabolic cost
- Data processing artifacts (neural spiking averaged over time and across trials)
- Absence of delayed activity in some experiments





Funahashi, Bruce, and Goldman-Rakic, 1989

Fig. 1. Average firing of two units during five delayed response trials with 30-second delays, 20 seconds of spontaneous activity record preceding each trial. (A) Unit in prefrontal cortex: (B) unit in nucleus medialis dorsalis (MD).







Short-Term Synaptic Plasticity (STP)

A pioneering study revealed that the interactions among pyramidal neurons in the PFC display synaptic facilitation lasting hundreds of milliseconds [Wang et al., Nature Neuroscience (2006)]:

Synaptic Plasticity could be relevant for WM

Short-term synaptic plasticity can be mimicked with a model developed by Tsodyks, et al. (1998) based on 2 synaptic variable :

depression x(t) and facilitation u(t)

The post-synaptic potentials delivered by neuron *i* are given in this model by:

 $Jx_i(t)u_i(t)$

 $x_i(t)$ is the fraction of still available resources after neurotransmitter depletion

 \mathbf{z}_i is the the fraction of available resources x_i ready for use : $u_i x_i$



Short-Term Synaptic Plasticity (STP)

The evolution in time of these 2 synaptic variables is given by

$$\frac{dx_i}{dt} = \frac{1 - x_i}{\tau_d} - u_i x_i \delta(t - t_k^{(i)}) \qquad \frac{du_i}{dt} = \frac{U - u_i}{\tau_f} + U(1 - u_i)\delta(t - t_k^{(i)})$$

Following a spike emission,

 u_i increases due to spike-induced calcium influx to the presynaptic terminal

after which a fraction u_i of available resources x_i is consumed to produce the post-synaptic current

Between spikes,

- u_i decays back to U with time constant au_f
- $lacksim x_i$ recovers to one with time constant au_d







Short-Term Synaptic Plasticity (STP)

Facilitation Dominated Synapse $\tau_f = 1500 \text{ ms} >> \tau_d = 200 \text{ ms}$



[Tsodyks, Pawelzik, Markram (1998) Neural Computation]



Synaptic Theory of Working Memory

A new paradigm

- A spiking network model for WM with synaptic depression x and facilitation u
- Cue presentation triggers: depression x & facilitation u
- Depression: Triggers population bursts needed to refresh memory
- Facilitation: Silent WM maintenance & selectivity for unspecific stimuli

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[Mongillo, Barak, and Tsodyks,
Science (2008)]
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Synaptic Theory of Working Memory

Benefits of Synaptic Theory of Working Memory

- WM is maintained in absence of spiking
- Population bursts allow interference free storage of more memory items
- Information is stored at a population level

We search for a neural mass model for WM:

- Based on short-term synaptic plasticity (STP)
- Able to exhibit spike synchrony
- Capable to give acces to experimental measures (EEGs,LFPs,ERPs)

IDEA : extend the MPR model by including STP





Neural Mass with Synaptic Depression and Facilitation

Short-term synaptic plasticity (STP):



- Depression⁴ : $\dot{x} = \frac{1-x}{\tau_d} uxr$
- Facilitation⁴: $\dot{u} = \frac{U-u}{\tau_{\rm f}} + U(1-u)r$

On population level: macroscopic STP

Neural mass 4 equations: $\tau_{\rm m}\dot{r} = \frac{\Delta}{\pi\tau_{\rm m}} + 2rv$ $\tau_{\rm m}\dot{v} = v^2 - (\pi\tau_{\rm m}r)^2 + Jux\tau_{\rm m}r + \bar{\eta}$ $\dot{x} = \frac{1-x}{\tau_{\rm d}} - uxr$ $\dot{u} = \frac{U-u}{\tau_{\rm f}} + U(1-u)r$



INS, Marseille 01/04/21 - p. 32





- Each Item is Loaded in an Excitatory Population
- The Inhibitory Pool avoids Abnormal synchronization
- 3 Neural mass Models ===> 10 Degrees of Freedom
- Comparison with networks with N = 600,000 neurons



Loading of One Item in the WM





Comparison with Experimental Data

The novelty of this neural mass is that we have at disposal the mean membrane potential V and we can estimate the Power Spectra (the Spectrogram) and compare with experimental measurements at a macroscopic scale (LFPs, EEGs, ERPs)



LFP measurements in prefrontal cortex of monkeys during WM tasks



LFPs in Monkeys







LFPs in Monkeys

Simulation resembles experimental results:

- δ -band (2 Hz 4 Hz) activity locked to stimulus onset
- Sustained β -band (12 Hz 25 Hz) activity



Beyond the experiment

- Encoding via PING like mechanism in $\beta\text{-}$ band
- Excitatory population bursts generate δ band signal



EEGs in Humans







Multi-item Loading in WM

Multiple items can be loaded in an architecture with 7 excitatory populations and 1 inhibitory pool

Three Item Loading in WM

- Activity in the δ -band upon item presentation
- Fundamental memory cycle frequency f_{cycle}
- Frequency of burst emission *f*_{burst}
- Resonance with the β -band sub-thresold oscillations





Working Memory Capacity

A series of studies have investigated the working memory capacity and indicated as maximal number of stored items $N_c\simeq 3-5$



[Cowan N, Behavioral and brain sciences (2001); Cowan N, Current directions in psychological science (2010)]



Working Memory Capacity

How to measure the WM Capacity from neurophysiological data ? Vogel et al. (Nature 2004, Nature 2005) introduced a measure of the WM capacity on humans based on event-related potentials (ERPs) from adults performing a visual memory task.



The task consists in memorizing an array of N_L colored squares



Working Memory Capacity

The mean membrane potential can be employed, analogously to the ERP in the experiments, as a proxy to measure the memory load and capacity





Neural Mass for Fluctuation Driven



Populations

- The MPR Neural Mass (PRX,2015) reproduces the dynamics of one heterogeneous population of globally coupled neurons
- The MPR model can be extended to two or more interacting populations
- The MPR model can be extended to the whole connectome: [V. Jirsa, S. Olmi, G. Rabuffo, et al, bioRxiv 2 preprints (2021)]
- The MPR model has been extended to encompass delay, gap junctions, short-term plasticity, asymmetric spike forms, conductance based neurons etc

However so far this Neural Mass always concerns globally coupled neurons without noise sources, a non realistic representations of neural systems, which always present:

background noise

random distribution of the synaptic connections

May we develop a Neural Mass encompassing quenched and dynamical disorder sources ?

YES WE DID !

Neural Mass for Fluctuation Driven Populations



- The MPR model is based on the assumption that the distribution of the membrane potentials is Lorentzian (LD),
- The presence of dynamical disorder modifies the LD, which is now distorted
- The LD cannot be expanded in regular cumulants or moments, they all diverge

We have introduced an expasion of the LD in pseudo-cumulants, to treat distorted LD

This allow to derive a neural mass encompassing different sources of noise

- A low dimensional mean-field model reproducing the dynamics of spiking QIF neurons
 - subject to background noise
 - and/or arranged in sparse random network

[Goldobin, diVolo, AT, "A reduction methodology for fluctuation driven population dynamics", preprint (2020)]



The Model

For a heterogeneous population of QIF neurons subject to different noise sources we can derive a 4 dimensional neural mass model

$$\dot{r} = \frac{\Delta_{\eta} + \Delta_{J}r + p_{2}}{\pi} + 2rv$$
$$\dot{v} = I_{0} + \eta_{0} + J_{0}r - \pi^{2}r^{2} + v^{2} + q_{2}$$
$$\dot{q}_{2} = 2\mathcal{N}_{R} + 4(q_{2}v - \pi p_{2}r)$$
$$\dot{p}_{2} = 2\mathcal{N}_{I} + 4(\pi q_{2}r + p_{2}v)$$

Four Macroscopic Variables :

Mean Membrane Potential and Firing Rate (v, r)

Two Dynamical Variables for the LD Distortions (q_2, p_2)

Quenched heterogeneities

Excitabilities : LD with (η_0, Δ_η)

Synaptic Couplings : LD with (J_0, Δ_J)

Noise Sources are encompassed in the terms $(\mathcal{N}_R, \mathcal{N}_I)$



Background Noise

Globally coupled inhibitory network of QIF neurons each subject to independent additive Gaussian noise of variance σ^2 : ($N_R = \sigma^2$, $N_I = 0$)



- the bifurcation diagram reveals Asynchronous States (ASs), Collective Oscillations (COs) and regions of coexistence of ASs and COs
- the neural mass results are in agreement with the network simulations
- the MPR model cannot capture even qualitatively these regimes displayed by the noisy spiking network
- Noise induced COs in absence of any synaptic or delay time scale

[Goldobin, diVolo, AT, bioRxiv (2020)]



Sparse Networks

- Random network with in-degrees k_j distributed as a LD, with median K and HWHM $\Delta_K = \Delta_0 K$.
- We assume that each neuron receive k_j independent Poissonian spike trains with rate r

This amounts to set $(\mathcal{N}_R = (J_0^2 r)/(2K), \mathcal{N}_I = -\Delta_0 \mathcal{N}_R)$





Summary

The Next Generation Neural Masses of MPR open a complete new perspective for realistic simulations of heterogenous spiking networks

- The Neural Masses reproduce with high fidelity the network dynamics:
 - not only the firing rate but also the sub-threshold membrane potential dynamics
 - synchronization and de-synchronization phenomena
 - The Neural Masses reproduces relevant neuroscience phenomena:
 - θ - γ cross coupling in the hippocampus and other areas
 - fast and slow γ oscillations in the hippocampus
 - working memory processes
- the neural mass MPR can be extended to capture fluctuation driven phenomena present in realistic brain circuits



Collaborators



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Fundings & Publications

- M. di Volo, AT, "Transition from asynchronous to oscillatory dynamics in balanced spiking networks with instantaneous synapses", Phys. Rev. Lett. (2018)
- H. Bi, M. Segneri, M. di Volo, AT "Coexistence of fast and slow gamma oscillations in one population of inhibitory spiking neurons", Physical Review Research (2020)
- A.Ceni, S. Olmi, AT, D. Angulo Garcia, "Cross frequency coupling in next generation inhibitory neural mass models", Chaos (2020)
- M. Segneri, H.Bi, S. Olmi, AT, "Theta-nested gamma oscillations in next generation neural mass models", Frontiers in Computational Neuroscience (2020)
- H. Taher, AT, S. Olmi, "Exact neural mass model for synaptic-based working memory", PLOS Computational Biology (2020)
- D. Goldobin, M diVolo, AT, "A reduction methodology for fluctuation driven population dynamics", bioRxiv (2020)