Synchronization

Alessandro Torcini

alessandro.torcini@u-cergy.fr

LPTM

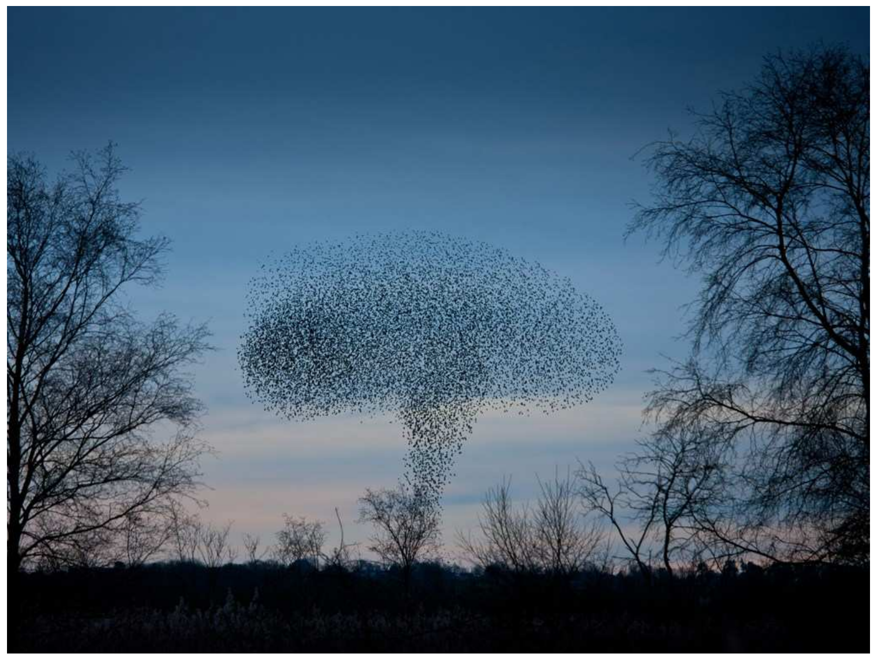
Université de Cergy-Pontoise

Complex Systems

A possible definition

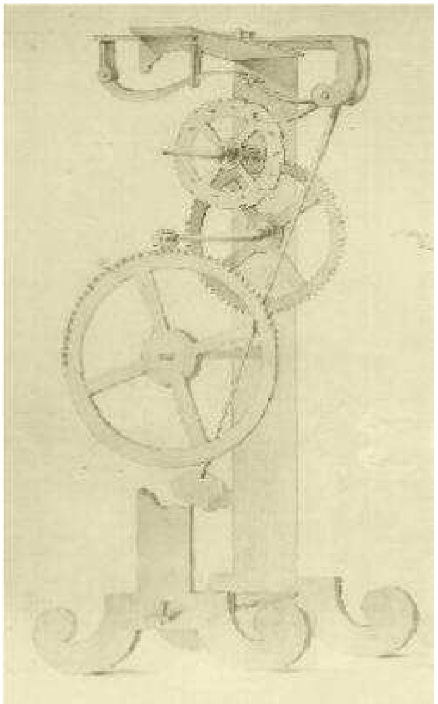
Complex systems are those composed of a large number of interacting elements, so that the collective behaviour of those elements goes far beyond the simple sum of the individual behaviours.

- schools of fishes
- swarm of birds



A simple element

Interesting collective motions arise also for simpler elements than a bird, let us consider the most regular object one can imagine: a clock !!!



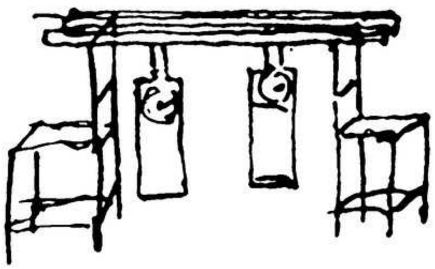
Galileo Gailei was the first who had the idea to exploit the regularity of pendulum oscillations to realize a clock, however was the Dutch scientist Christian Huygens to realize it in 1656.

The first clock had an error less than 1 minute per day, an incredible good accuracy at the time.

The regular oscillations of a pendulum can be described mathematically by only one variable describing the pendulum position : a phase variable θ.

Synchronization of Two Clocks

Christiaan Huygens reported the first observation of synchronization:



"... It is quite worth noting that when we suspended two clocks so constructed from two hooks imbedded in the same wooden beam, the motions of each pendulum in opposite swings were so much in agreement that they never receded the least bit from each other and the sound of each was always heard simultaneously. Further, if this agreement was disturbed by some interference, it reestablished itself in a short time. For a long time I was amazed at this unexpected result, but after a careful examination finally found that the cause of this is due to the motion of the beam, even though this is hardly perceptible."

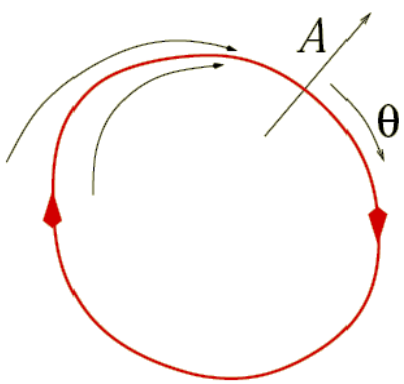
Antiphase Synchronization

This problem is still nowdays studied:
Bennett, Schatz, Rockwood, Wiesenfeld,
"Huygens Clocks", Proc. R. Soc. Lond. A, vol. 458 (2002), pp. 563 - 579.

Phase Model I

Many physical, chemical, biological systems exhibit Rhytmic Oscillations

- A.T Winfree, The geometry of biological time (2001)
- G. Buzsaki, Rhythms of the Brain (2006)



A system exhibiting a stable periodic motion γ is called an Oscillator, we will study synchronization properties of these systems.

The motion can be characterized by the time t from the last crossing t_n of a certain point x_0 on the orbit, and a phase can be introduced as

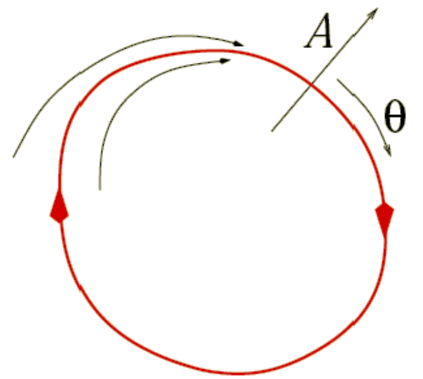
$$\theta = \frac{t - t_n}{t_{n+1} - t_n} 2\pi \qquad 0 \leq \theta \leq 2\pi$$

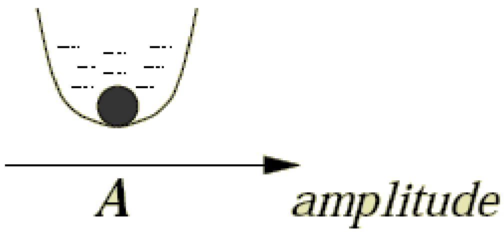
the dynamics on the orbit can now be rewritten simply as $\boxed{\dot{\theta} = \omega_0}$ where ω_0 is the natural frequency of the oscillation.

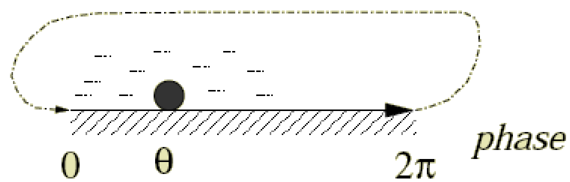
Information on the amplitude oscillation (on the radius of the orbit) is lost, but not on its phase.

Phase Model II

- $\dot{\theta} = \omega_0$ $\lambda = 0$ (Phase is Marginally Stable)
- $\dot{A} = -\eta(A - A_0)$ $\lambda = -\eta$ (Amplitude (radius) is Stable)







- Since the amplitude (radius) is stable it is difficult to modify it with small perturbations
- The phase is at the edge between stability and instability small perturbations (due to external forcing or coupling) can induce large modifications of the phase

Thus with a small forcing it is possible to adjust the phase and the frequency of the oscillations, without altering the amplitude:

this is the essence of the synchronization phenomenon

Everyone likes to synchronize

Tout le monde aime synchroniser

- Les Lucioles
- Danseurs et Danseuses
- et aussi . . . les oscillateurs

Collective synchronization

Populations of biological oscillators can spontaneously synchronize to oscillate with a common frequency, despite a distribution of different natural frequencies among the population

- swarms of fireflies flash in synchrony;
- crickets chirp in unison;
- groups of women whose menstrual cycles synchronize.

Summary

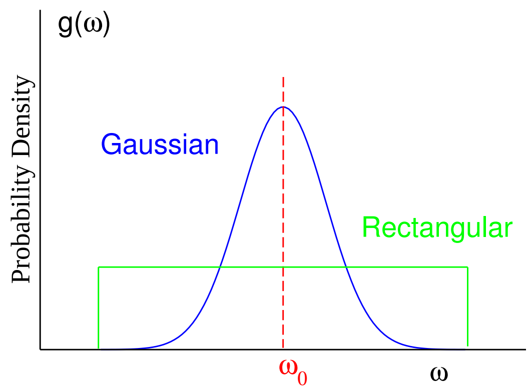
- The Kuramoto Model
- The Millennium Bridge
- Neuronal Synchronization
- Chimera States
- Exact Reduction Methods

Kuramoto Model

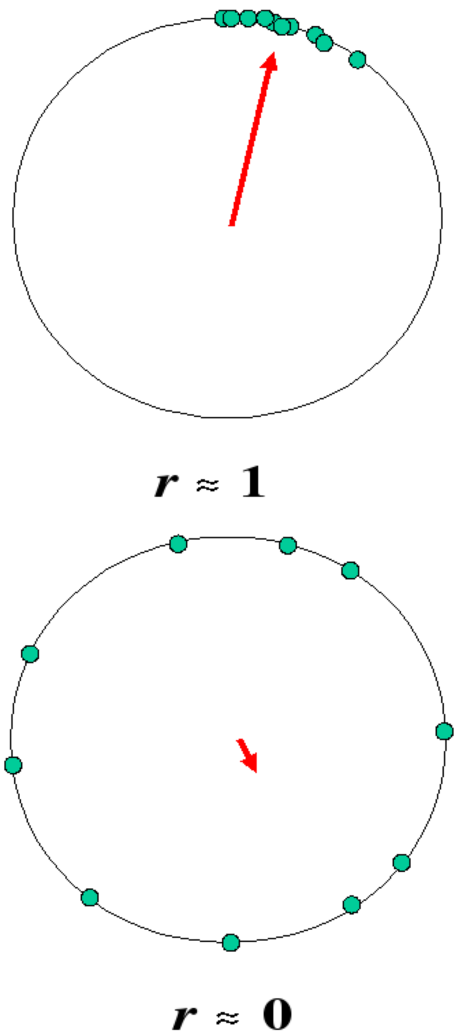
- N coupled phase oscillators with different frequencies ω_k
- Frequencies distributed according to $g(\omega)$

$$\frac{d\theta_k}{dt} = \omega_k + \frac{k}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_k)$$

- The coupling is rescaled by N to avoid divergence of the forcing term in the thermodynamic limit ($N \to \infty$)



Synchronization Parameter



Mean Field Variables Amplitude R and Phase Θ

$$Z = R\mathrm{e}^{i\Theta} = \frac{1}{N}\sum_{k=1}^{N} \mathrm{e}^{i\theta_k}$$

$R = |Z|$ is a Synchronization Indicator

- if $\theta_k = \theta_j \quad \forall j, k$ System Fully Synchronized $R \equiv 1$
- if θ_k are equally distributed on the circle
  Desynchronized System $R \simeq \frac{1}{\sqrt{N}}$
- If some oscillator are frequency locked $R \neq 0$

The Kuramoto model can be rewritten as

$$\frac{d\theta_k}{dt} = \omega_k + kR\sin(\Theta - \theta_k)$$

each oscillator is forced by the Self-Consistent Mean Field Z

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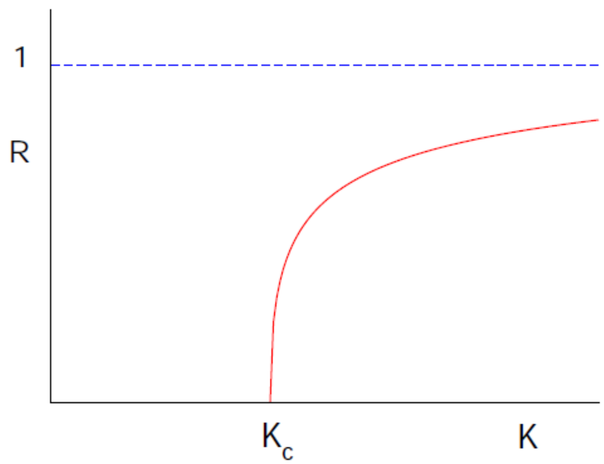
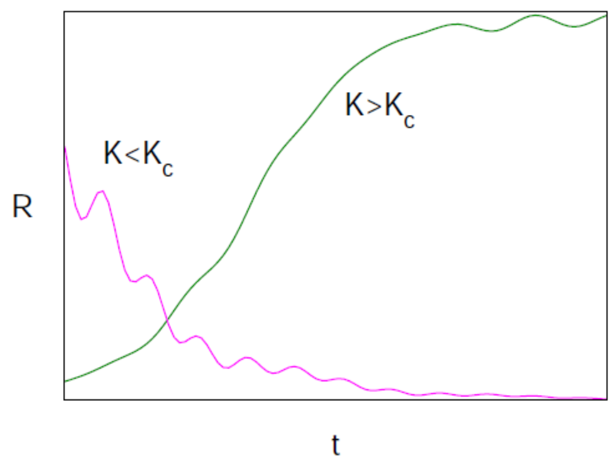
Synchronization Transition

We observe a continuous transition from the incoherent to the coherent regime at k_c,





- Incoherence: all the oscillators run at their natural frequencies ($k < k_c$ and $R \sim 0$)
- Partial Locking: some of the oscillators are locked, while the others drifts at different frequencies ($k > k_c$ and $R \neq 0$)
- Complete Locking : all the oscillators are locked , the phase difference between any two oscillators is constant in time ($k >> k_c$ and $R \sim 1$)

Millennium Bridge

Crowd Synchrony on the Millennium Bridge

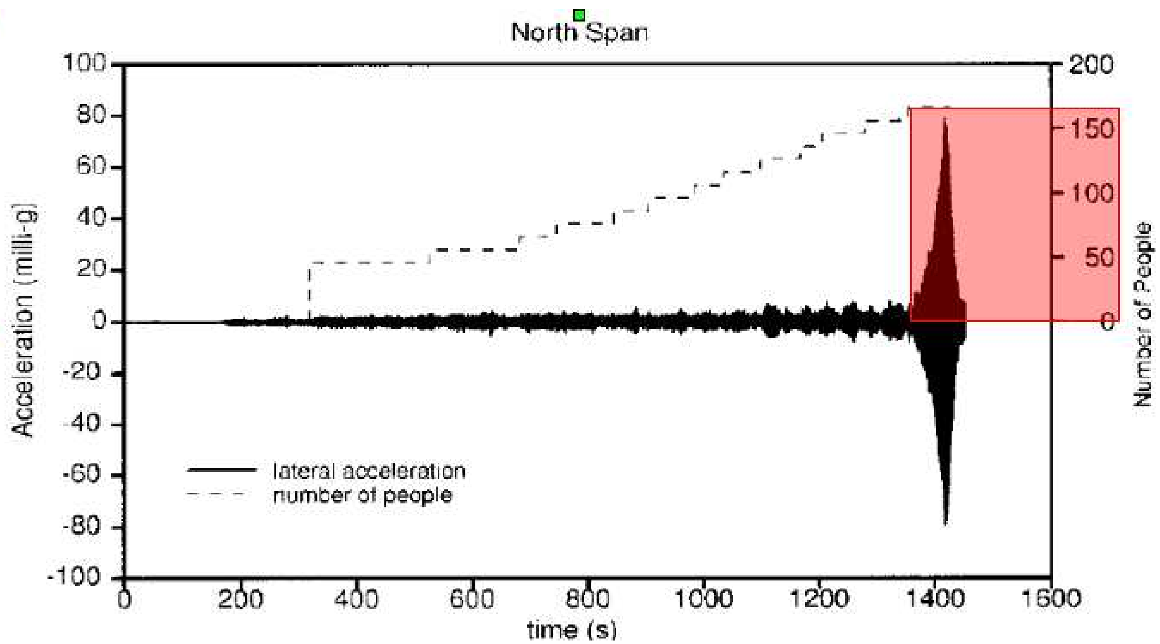
Strogatz, Abrams, Mc Robie, Eckhardt, Ott Nature, 438 (2005) 43

When the London Millennium Bridge opened on June 10, 2000, soon after the crowd streamed on the bridge, the bridge begins to oscillate from side to side (to wobble): many pedestrians synchronize spontaneously their steps with the bridge's vibrations, amplifying them. The synchronized steps of the people caused such heavy oscillations that the bridge had to close down until dampers were put in 2 years later.

Collective synchronization was responsible for the wobbling of the bridge

Millennium Bridge

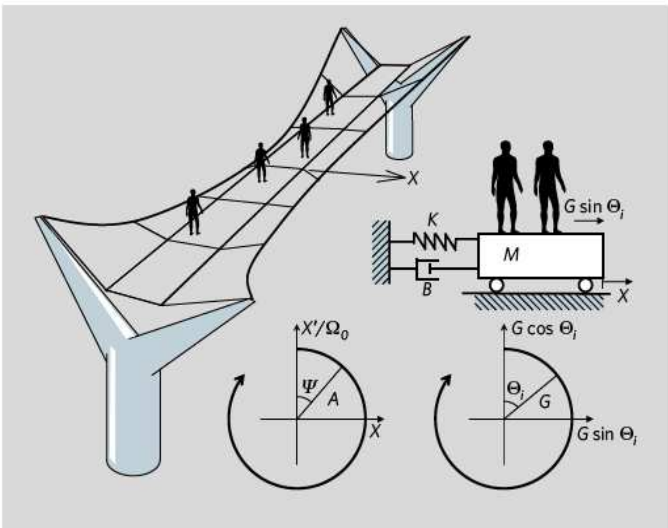
The experiment by Arup



Groups of people of increasing number walk together along the bridge until it begins to wobble, there is a critical number of people

Millennium Bridge

The Model



- The left/rigth walking cycle of each pedestrian is seen as an oscillator θ_i with his own frequency ω_i forced by the bridge oscillations;
- The lateral motion X of the bridge is schematized as a weakly damped harmonic oscillator driven by the collective motion of the pedestrians.

$$M\frac{d^2X}{dt^2} + B\frac{dX}{dt} + KX = G\sum_{i=1}^{N}\sin\theta_i \quad \text{where} \quad X = A\sin\psi$$

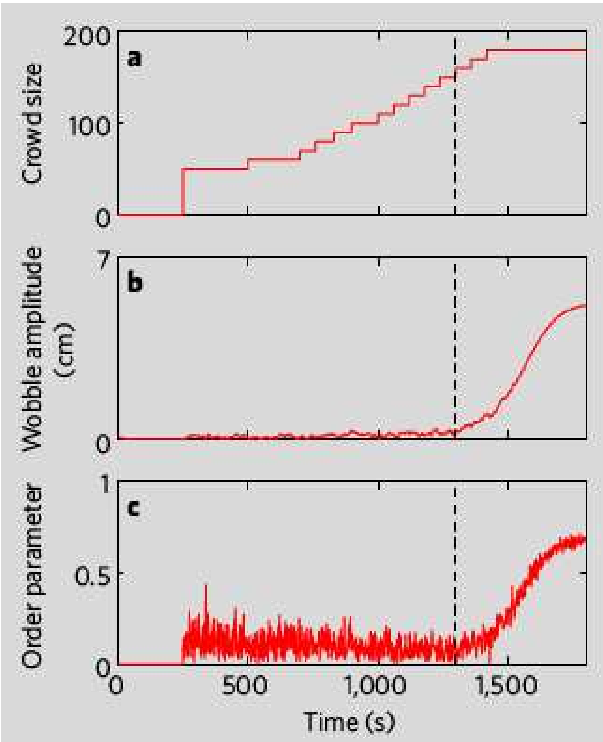
where M, B and K are the mass, the damping and the stifness associated to the lateral motion of the bridge and G is the maximal force exerted by a pedestrian

$$\frac{d\theta_i}{dt} = \omega_i + CA\sin(\psi - \theta_i + \alpha)$$

where C is the sensitivity of the pedestrian to bridge vibration, to be fitted

Millennium Bridge

The Simulation Result

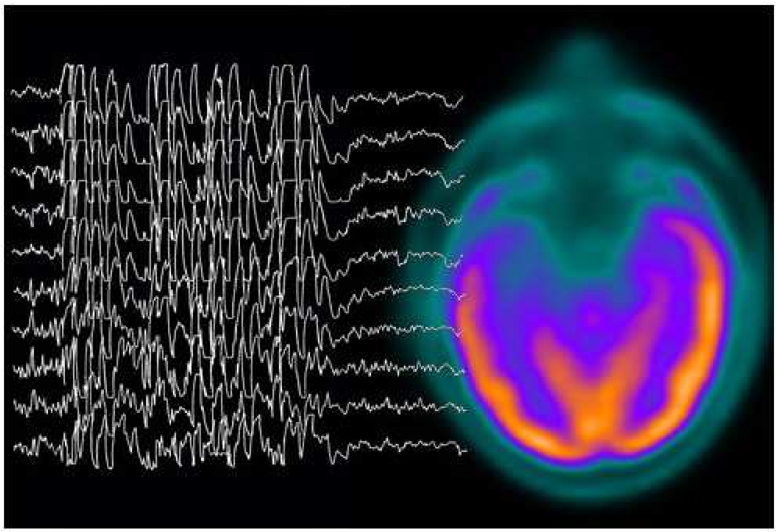


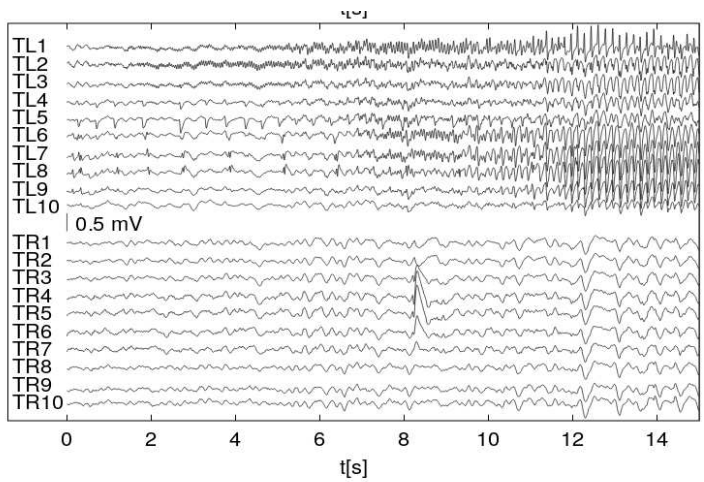
- The simulations have been performed by employing realistic values for the parameters, apart C which has been fitted to the experimental data by Arup
- The simulation start with bridge at rest and $N = 50$ pedestrians on the bridge
- The number of pedestians is increased by 10 at each step

Bernard Feldman, a writer for Physics Today, however, believes Strogatz is wrong: since the frequency of the lateral oscillation of bridges is around 0.5 Hz whereas the average frequency of walking is 1.0 Hz (2 steps per second). Therefore it is unlikely that synchronized footsteps could have intensified the wobbling of the Millennium Bridge.

Neuronal Synchronization

Neurons in the cortex usually fire in an asynchronous and sparse way, high level of synchronization are usually sign of functional brain disorder, e.g. Epilepsy .



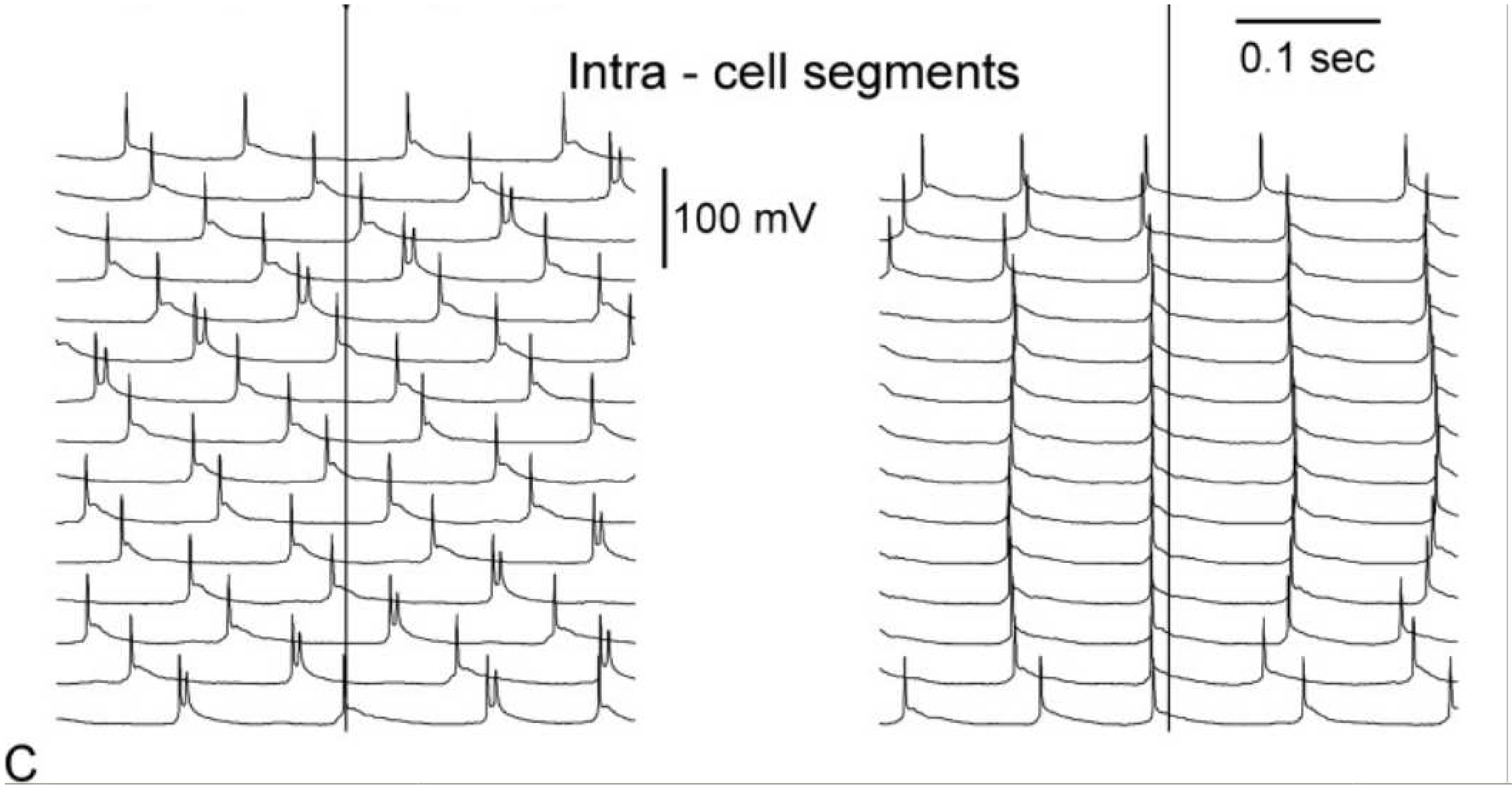


EEG displaying a seizure measured during an epileptic event.

Epilepsy is not exactly rare 1% of the worldwide population is affected, it is important to understand the mechanisms behind it

Neuronal Synchronization

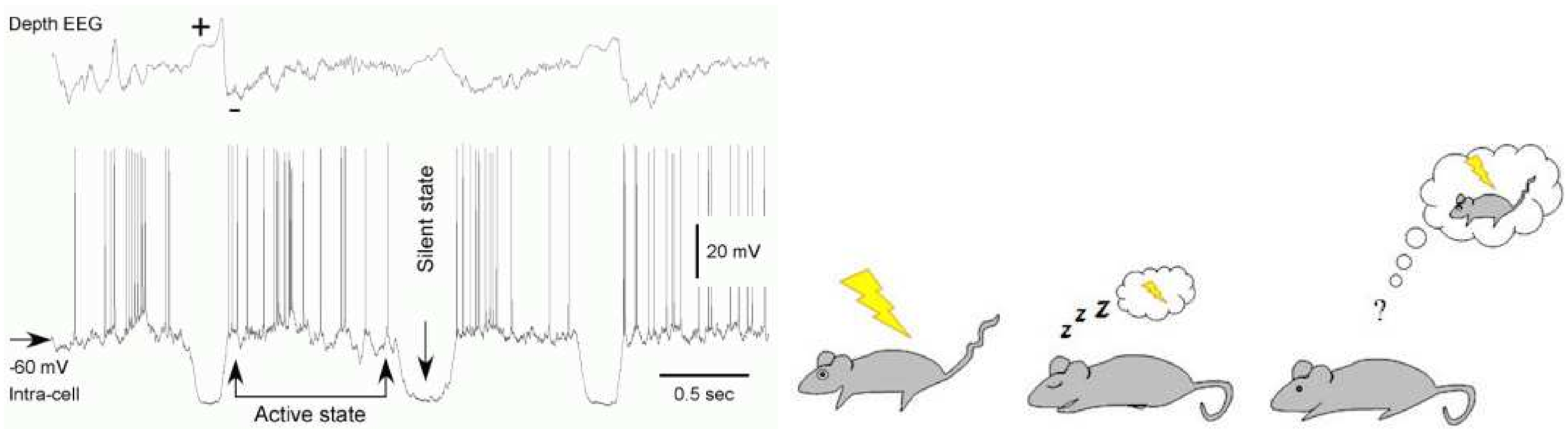
Neuronal synchronization seems a key aspect



Single neuron firing during an epileptic event.

Neuronal Synchronization

Neuronal synchronization can have positive effects. the phenomenon of slow wave sleep has been related to memory consolidation during the sleep.



Slow wave oscillations in the cortex

At EEG level, the slow oscillation appears as periodic alterations of positive and negative waves :

1. During EEG depth-positivity, cortical neurons remain in silent state.
2. During EEG depth-negativity cortical neurons fire action potentials.

It was shown that during slow-wave sleep neocortical and thalamic neurons display phase synchronization.

Chimera

La Chimera d'Arezzo

Etruscan Art - Museo Nazionale di Archeologia (Firenze)

In Greek mythology, Chimera was a monstrous fire-breathing female creature of Lycia in Asia Minor, composed of the parts of multiple animals: upon the body of a male lion with a tail that terminated in a snake's head, the head of a goat arose on her back at the center of her spine (Wikipedia)

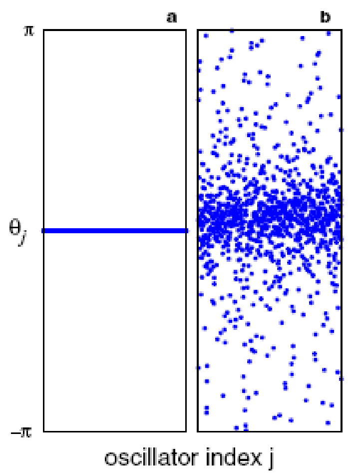
Chimeras in Oscillator Populations

Let us consider two oscillator populations $\{\theta^a\}$ and $\{\theta^b\}$ made of identical oscillators, where each oscillator is coupled to equally to all the others in its group, and less strongly to those of the other group

$$\frac{d\theta_i^a}{dt} = \omega + \frac{\mu}{N}\sum_{j=1}^{N}\sin(\theta_j^a - \theta_i^a - \alpha) + \frac{\nu}{N}\sum_{j=1}^{N}\sin(\theta_j^b - \theta_i^a - \alpha) \qquad \mu > \nu$$

Simulations of the 2 populations reveals two different dynamical behaviours

- Synchronized state $r = 1$
- A Chimera State: one population is synchronized and the other not



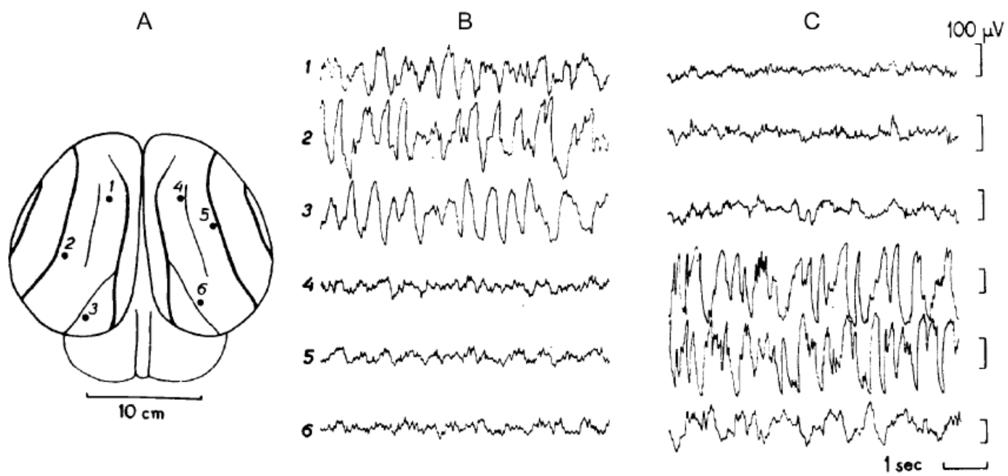
The oscillators are identical and symmetrically coupled : the Chimera State emerges from a spontaneous symmetry breaking

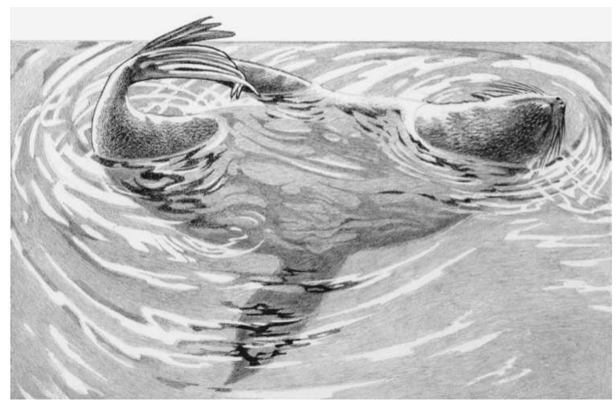
Abrams, Mirollo, Strogatz, Wiley, Phys. Rev. Lett 101 (2008) 084103

Unihemispheric sleep

Unihemispheric slow-wave sleep is the ability to sleep with one half of the brain while the other half remains alert. In contrast to normal sleep where both eyes are shut and both halves of the brain show reduced consciousness.

Among mammals, unihemispheric sleep is restricted to aquatic species (e.g. cetaceans). It is widespread in birds, and may even occur in reptiles. Unihemispheric sleep allows surfacing to breathe in aquatic mammals and predator detection in birds.





Unihemispheric slow wave sleep is characterized at a macroscopic level (EEG) by slow collective oscillations in the sleeping hemisphere and by asynchronous activity in the other hemisphere (broken spatial symmetry) [Rattenborg et al., Neuroscience & Biobehavioral Reviews (2000)]

Chimeras are Everywhere

Spontaneous symmetry breaking is a fundamental and universal phenomenon in complex systems exhibiting collective behavior, such as power grids, neuroscience, condensed matter systems, optomechanical crystals, or cells communicating via quorum sensing in microbial populations.

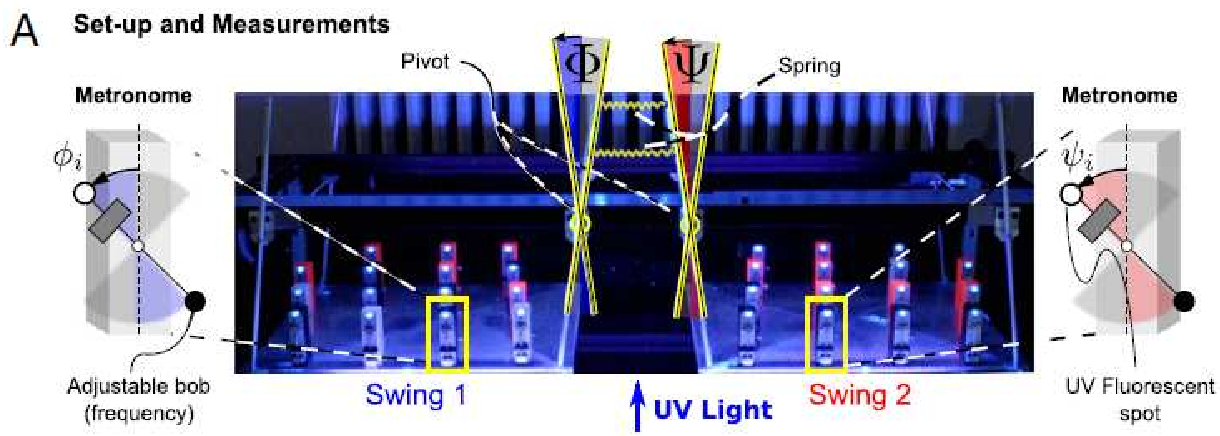
Experimental observations:

- opto-electronic devices: Hagerstrom,et al. , Nature Physics (2012).
- coupled chemical oscillators: Tinsley *et al.*, Nature Physics (2012);
- mechanical oscillator networks: Martens *et al.*, PNAS (2013); Olmi, Martens, Thutupalli, Torcini, Phys. Rev. E(R) (2015).
- electronic nonlinear delayed oscillators: Larger *et al.*, Phys. Rev. Lett. (2013).

as well as theoretical /numerical studies:

- I. Omelchenko,et al., Phys. Rev. Lett. (2013); G. Sethia, A. Sen, Phys. Rev. Lett. (2014); J. Sieber, O. Omel'chenko, M. Wolfrum, Phys. Rev. Lett. (2014); A. Yeldesbay, A. Pikovsky, M. Rosenblum, Phys. Rev. Lett. (2014).
- first evidence of chimeras in neural networks: Olmi, Politi, Torcini, EPL (2010).

The experiment



- Two populations of metronomes (self-sustained oscillator)
- Each population: $N = 15$ identical metronomes (same frequencies) on an alluminium swing (strong coupling)
- The two swings are coupled via 2 tunable springs (weak coupling)
- UV fluorescent spots on metronomes and swings

THE VIDEO !

E.A.Martens *et al.* PNAS , 2013

Exact Reduction Methods

For N phase oscillators whose dynamics is described by the Kuramoto model :

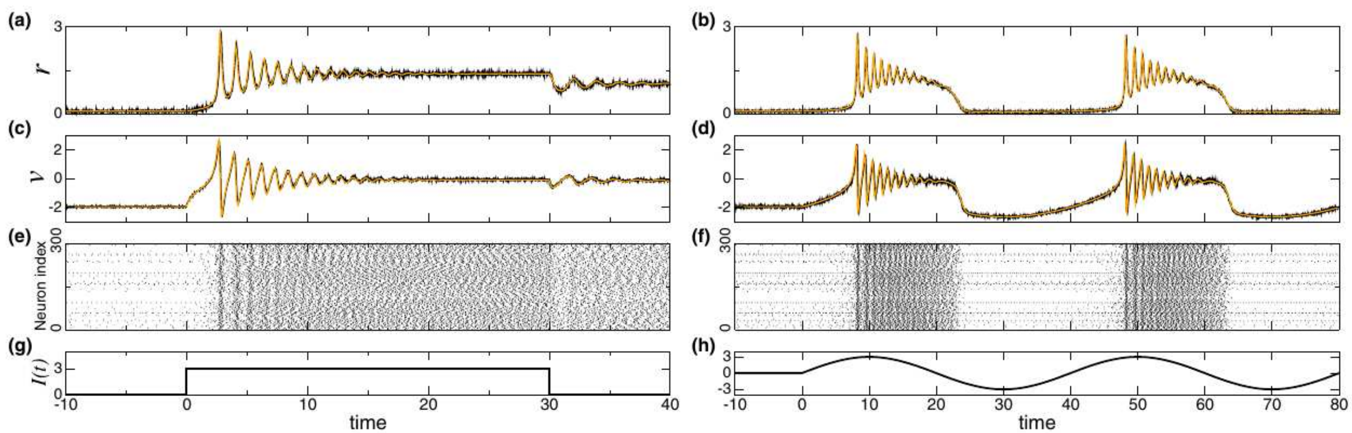
$$\frac{d\theta_k}{dt} = \omega_k + \frac{k}{N} \sum_{j=1}^{N} \sin(\theta_j - \theta_k) \quad k = 1, \ldots, N$$

the dynamics can be exactly rewritten in terms of few collective variables :

- Watanabe - Strogatz Physica D (1994) three variables for any finite N
- Ott - Antonsen, Chaos (2008) two variables in the limit $N \to \infty$

These results have been recently extended to

- Neural network models by Pazo-Montbrió Phys. Rev X (2014) and (2015)



Future events

Future events in Cergy-Pontoise on complex systems:

- Congrès international - Analytical and numerical insights on the dynamics of complex neural networks - (2018) Neurosciences et Complexité
- Semestre thématique sur la complexité - LABEX MMI-DDE (2017/18)

UNIVERSITÉ de Cergy-Pontoise

References

Texts employed for the lectures

- Chemical Oscillations, Waves, and Turbulence
  Y Kuramoto
- Synchronization: a universal concept in nonlinear sciences
  A. Pikovsky, M. Rosenblum, J. Kurths (Cambridge University Press, 2003)
- The Kuramoto model: a simple paradigm for synchronization phenomena
  JA Acebron, LL Bonilla, CJ Peréz Vicente, F Ritort, R Spigler Reviews of Modern physics 77 (2005) p. 137

Most popular texts

- Sync
  S. Strogatz (Hyperion Book, 2003)
- The geometry of biological time
  A.T. Winfree (Springer Verlag, 2001)
- Rhythms of the Brain
  G. Buzsaki (Oxford University Press, 2006)

Ott-Antonsen Ansatz

For certain systems of globally coupled phase oscillators the infinite dimensional dynamics is reduced to a flow on a phase space $\Rightarrow$ low dimensional dynamics

- Kuramoto model:

$$\frac{d\phi_i(t)}{dt} = \omega_i + \frac{K}{N}\sum_{j=1}^{N} \sin[\phi_j - \phi_i]$$

- The state of the oscillators system at time t can be described by a continuous distribution function $f(\omega, \phi, t)$ (or by $f^\sigma(\omega, \phi, t)$ with $\sigma = 1, \ldots, s$), where

$$\int_0^{2\pi} f(\omega, \phi, t) d\phi = g(\omega) \qquad or \qquad \int_0^{2\pi} f^\sigma(\omega, \phi, t) d\phi = g^\sigma(\omega)$$

  and $g(\omega)$, $g^\sigma(\omega)$ are time independent oscillator frequency distributions.

- In the thermodynamic limit the system is described by the density $f(\omega, \phi, t)$, where $f(\omega, \phi, t)g(\omega)d\omega d\phi$ gives the fraction of oscillators with natural frequency between ϕ and $\phi + d\phi$ at time t.

Ott-Antonsen Ansatz

- The Ott-Antonsen Ansatz is

$$f(\omega,\phi,t) = \frac{1}{2\pi}\left[1 + \sum_{n=1}^{\infty} \bar{\alpha}(\omega,t)^n e^{in\phi} + c.c.\right]$$

  - $f(\omega,\phi,t)$ constitutes an invariant manifold that determines the system's long-term dynamics;

- $f(\omega,\phi,t)$ obeys the following initial problem

$$\frac{\partial f}{\partial t} + \frac{\partial}{\partial \phi}(vf) = 0, \quad r = \int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\omega f e^{i\phi}$$

  where v is the r.h.s. of the Kuramoto model eq. and r is the order parameter

- Substituting the Ansatz in the initial problem, for integrable $g(\omega)$, the order parameter is simply α evaluated at a (complex) frequency and the system reduces to 2 ODEs for ρ, Φ, where $r = \rho e^{-i\Phi}$

[E. Ott, T. M. Antonsen, CHAOS 18, 037113 (2008)]