

# Clique of functional hubs in developmentally regulated neural networks

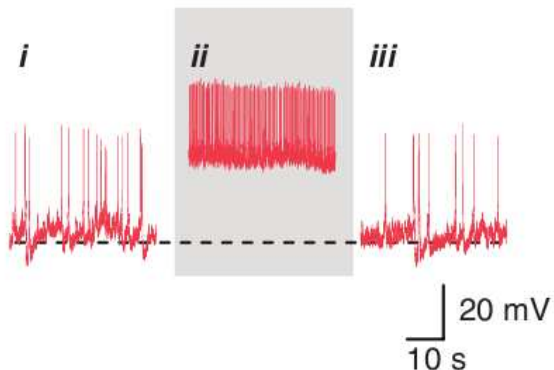
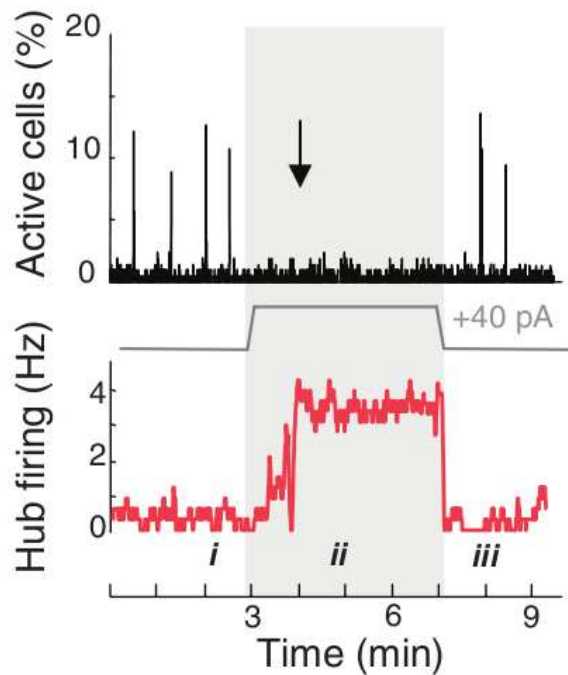
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<http://neuro.fi.isc.cnr.it/>



# Main Inspiration



Bonifazi *et al*, Science 2009

- Immature hippocampal circuits exhibit
  - Rhythmic collective oscillations
  - giant depolarizing potentials
  - Excitatory action of GABAergic transmission
  - Presence of functional and structural hub neurons
- Single neuron stimulation can silence the network activity - GABAergic hub interneurons

General questions

- To which extent a single neuron can influence brain circuits / network dynamics ?
- Why only few neurons displays such a strong power ?

## Neural Network Model:

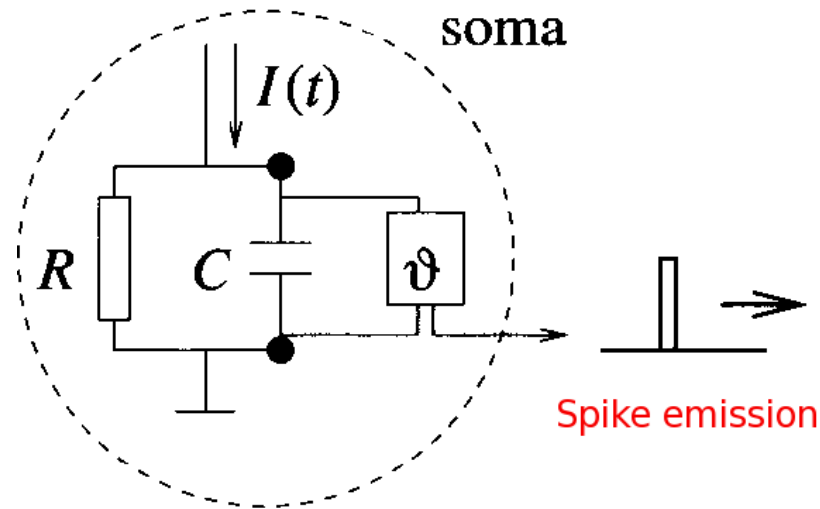
- Network of **Leaky Integrate-and-Fire neurons** with **Short-Term Synaptic Plasticity**
- Developing neuronal circuit → **Excitatory cells**
- Topology → **Random (Erdős-Renyi) network** (no scale free topology)
- **Key Ingredients:**
  - **Developmentally regulated constraints** on single neuron excitability and connectivity (Correlations)
- Dynamics
  - **Population Bursts** - Collective rhythmic oscillations

## Results:

- Few **critical neurons** control the population activity - **Functional Hubs**
- Critical neurons are arranged into a **clique**

# The Model

# Leaky-integrate-and-fire neuron



$$\tau_m \dot{V}(t) = -V(t) + I(t) \quad t < t_f \quad (\tau_m = RC)$$

$$V(t_f^-) = \theta \quad \rightarrow \quad \text{spike emission} \quad \delta(t - t_f)$$

$$V(t_f^+) = V_r \quad \rightarrow \quad \text{resetting mechanism}$$

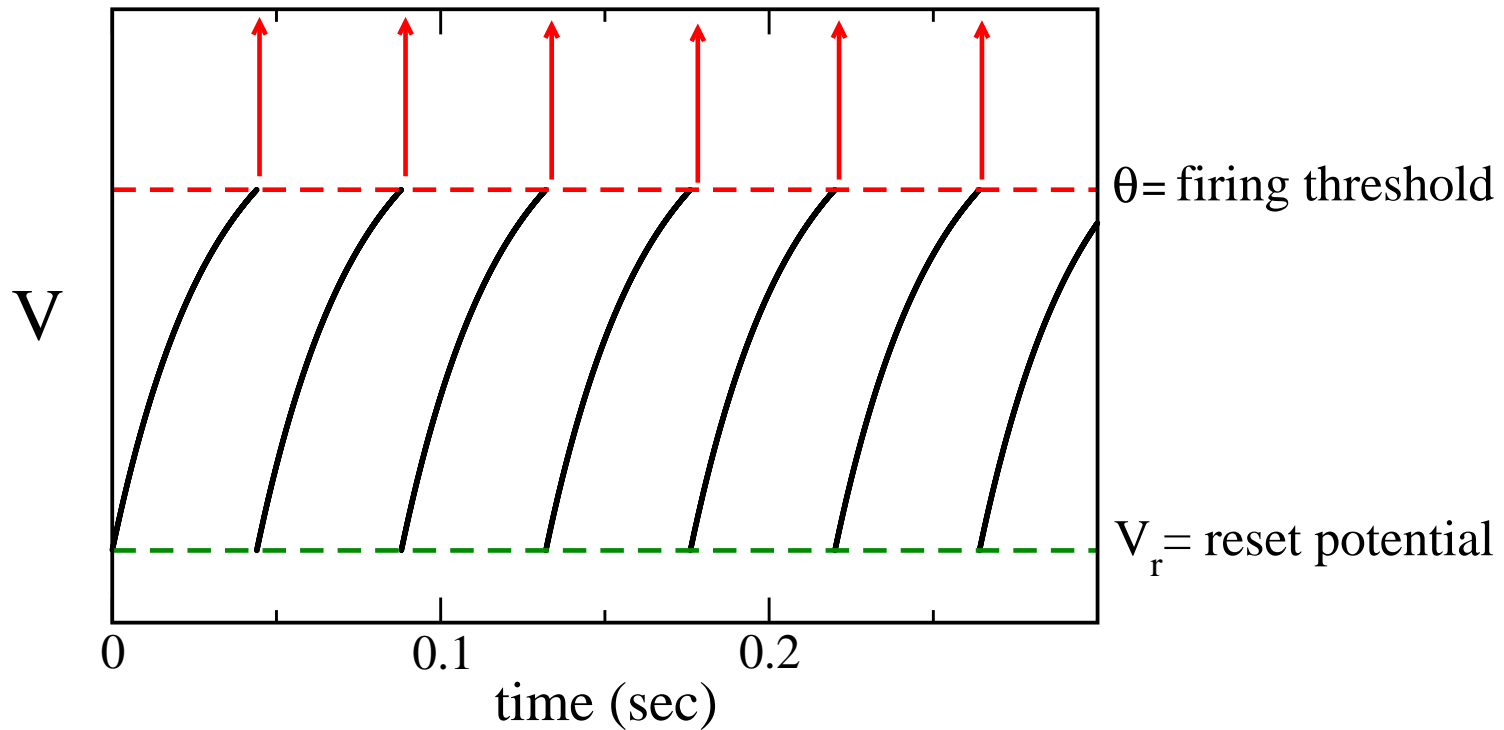
Parameters:  $\tau_m = 30$  ms,  $V_r = 13.5$  mV,  $\theta = 15$  mV

# Leaky-integrate-and-fire neuron



Response to a constant synaptic current

$$I(t) = I^b$$



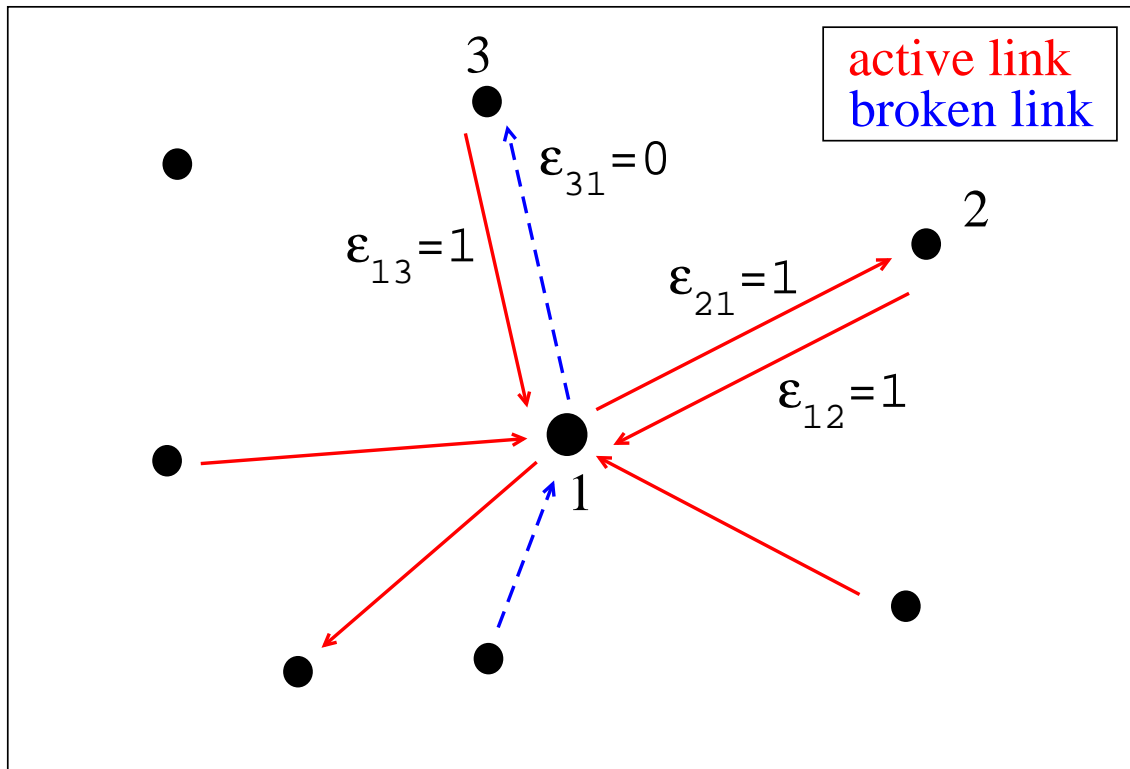
$$I^b > \theta$$

---> periodic firing

$$I^b < \theta$$

---> silent neuron (no spikes!)

# Directed Erdős-Rényi network



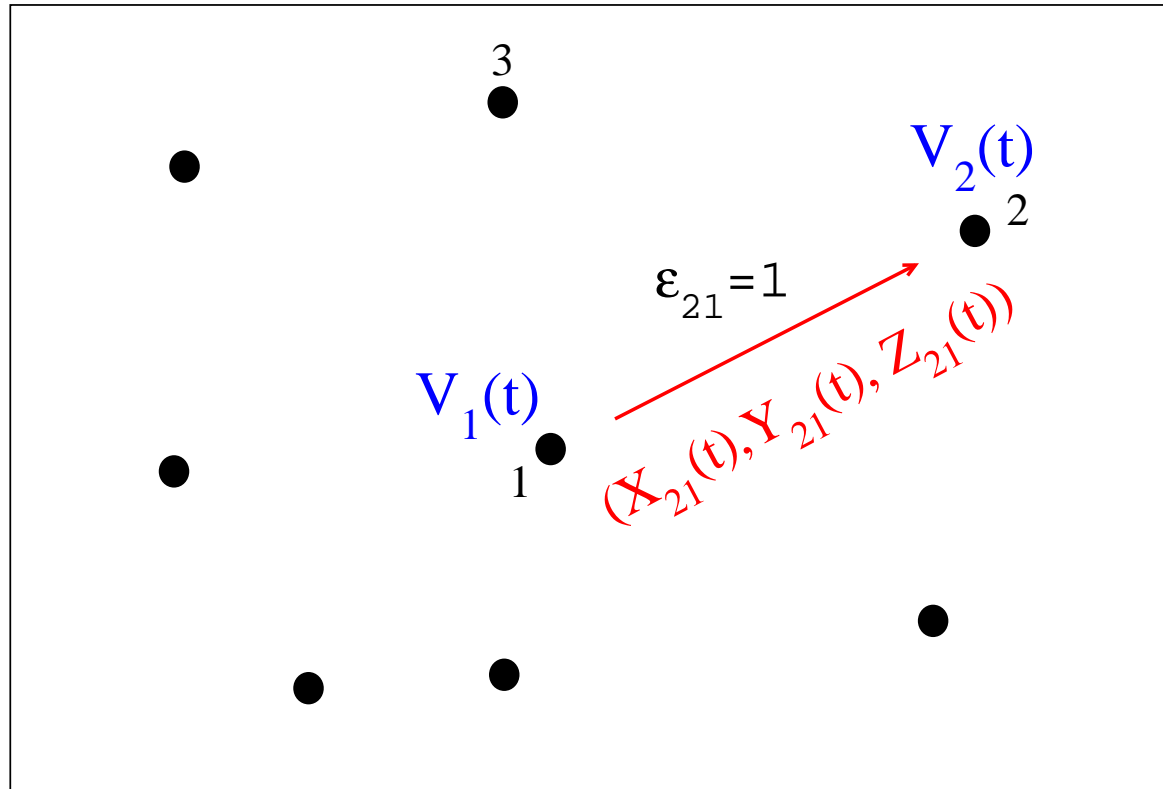
- $K_i^I$  Input connectivity
- $K_i^O$  Output connectivity
- $K_i^T = K_i^I + K_i^O$   
Total connectivity
- On average  
 $K^I = K^O = p \times N$

connection probability --->  $p = 10\%$

connectivity matrix --->  $\epsilon_{ij}$

# Short-term synaptic plasticity

Tsodyks, Uziel, Markram (J. Neurosc. 2000) → T.U.M. model



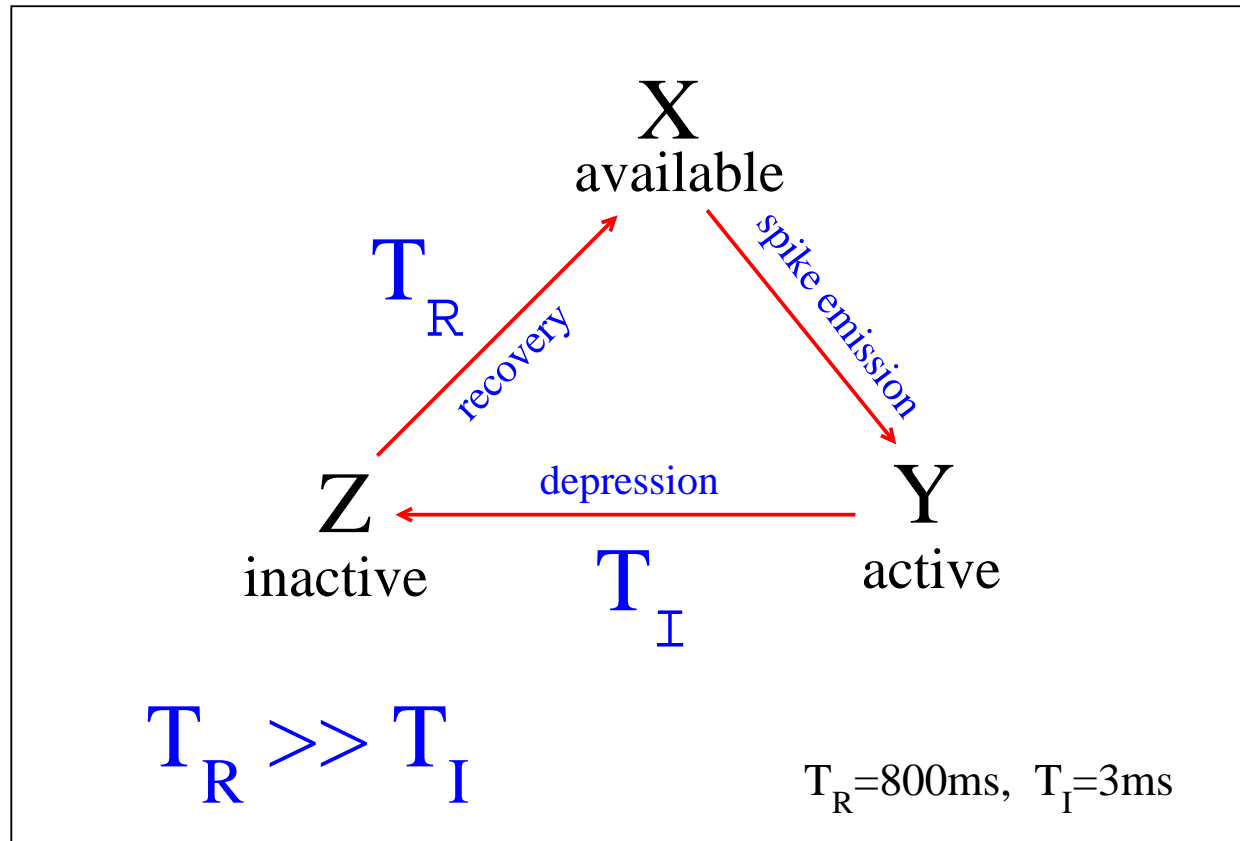
$\epsilon_{ij}$  ----> connectivity matrix

Mongillo, Barak, Tsodyks, Synaptic Theory of working memory, Science (2008)



# Short-term synaptic depression

Dynamics of the synaptic resources



$$X + Y + Z = 1$$

# The Complete Model



Network of N leaky integrate-and-fire neurons with short-term plasticity

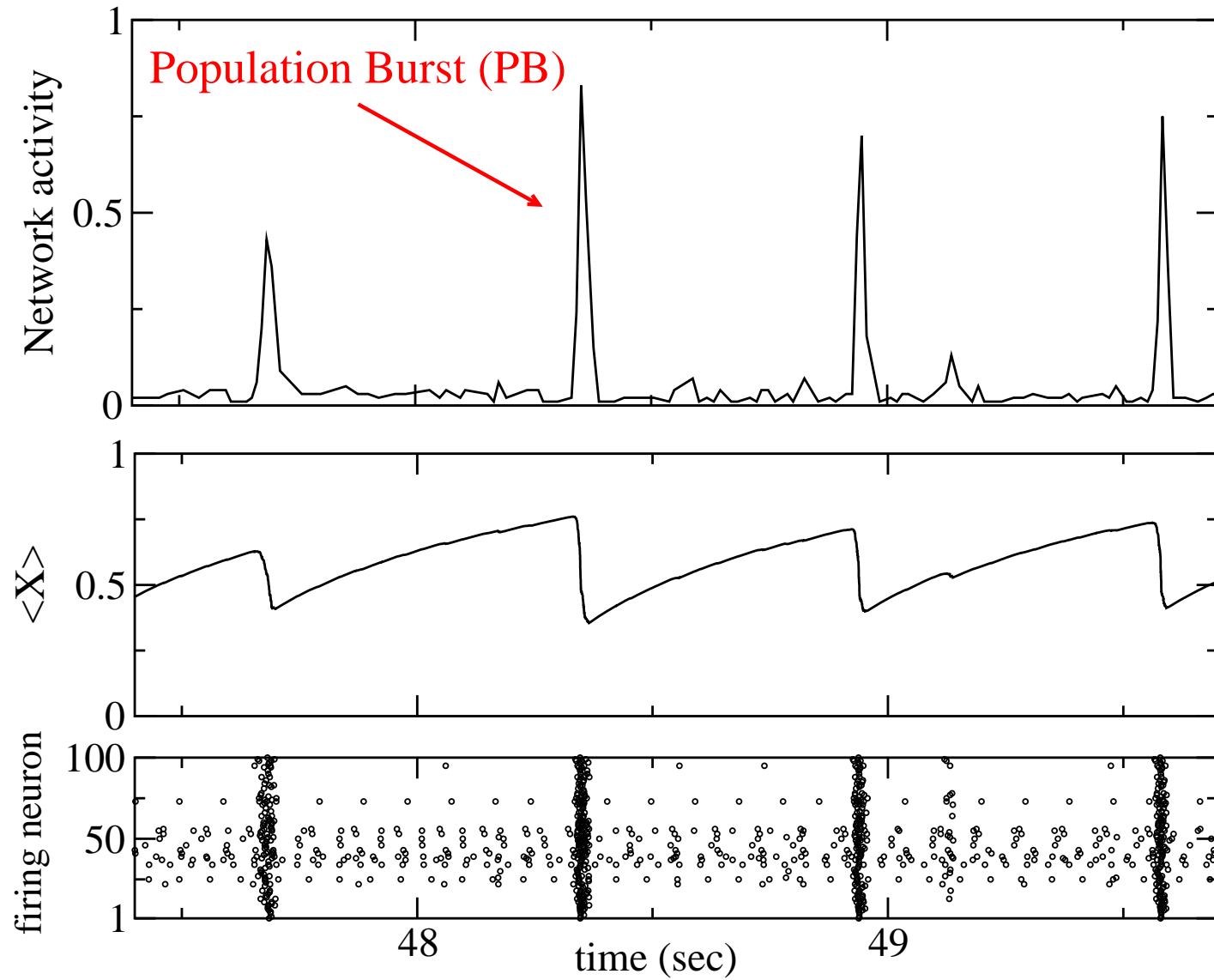
$$\tau_m \dot{V}_i = -V_i + I_i^b + \frac{G_i}{K_i^I} \sum_{j \neq i} \epsilon_{ij} Y_{ij} \rightarrow \text{synaptic strengths modulated by } Y_{ij}$$

$$\dot{Y}_{ij} = -\frac{Y_{ij}}{T_{ij}^I} + u_{ij} X_{ij} \sum_m \delta(t - t_j(m)) \rightarrow \text{spike train}$$

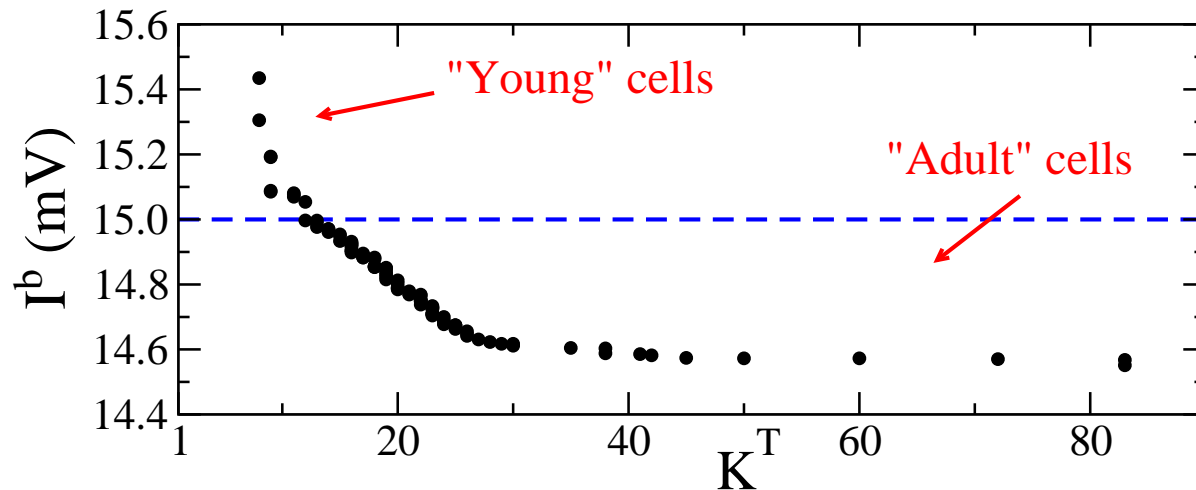
$$\dot{Z}_{ij} = \frac{Y_{ij}}{T_{ij}^I} - \frac{Z_{ij}}{T_{ij}^R} \quad (X_{ij} + Y_{ij} + Z_{ij} = 1)$$

- **Excitatory** Neurons –  $G_i > 0$
- $I_i^b$  = **intrinsic excitability**  $\rightarrow$  if  $I_i^b > \theta$  neurons can fire even if isolated !
- The currents are measured in mV, since they incorporate the membrane input resistance

# Typical dynamics



# Developmentally regulated network



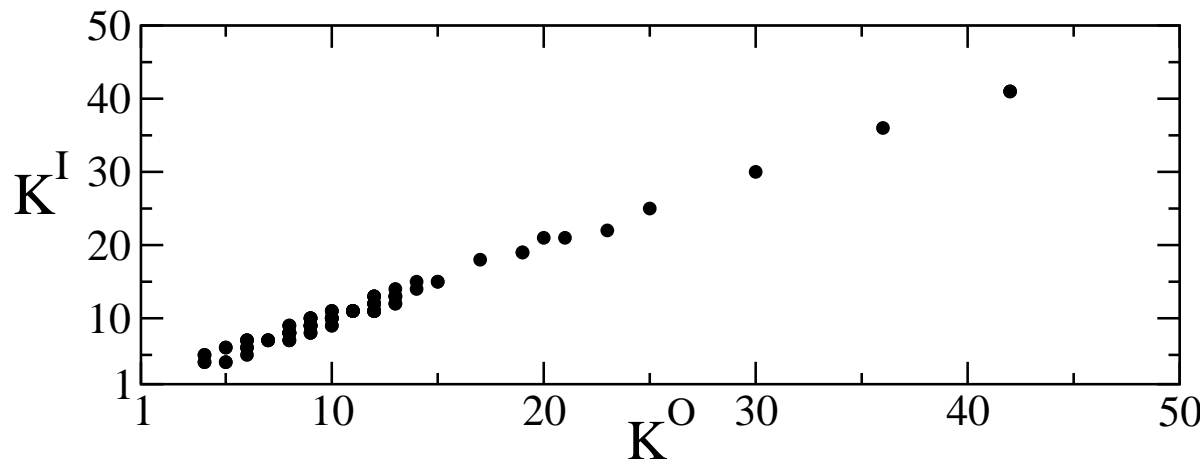
Our hypothesis:

$\theta$

Young (adult) cells have low (large) number of synapses  $K^T$

Young cells are more excitable than adult ones

Number of afferent  $K^I$  and efferent  $K^O$  synapses are proportional

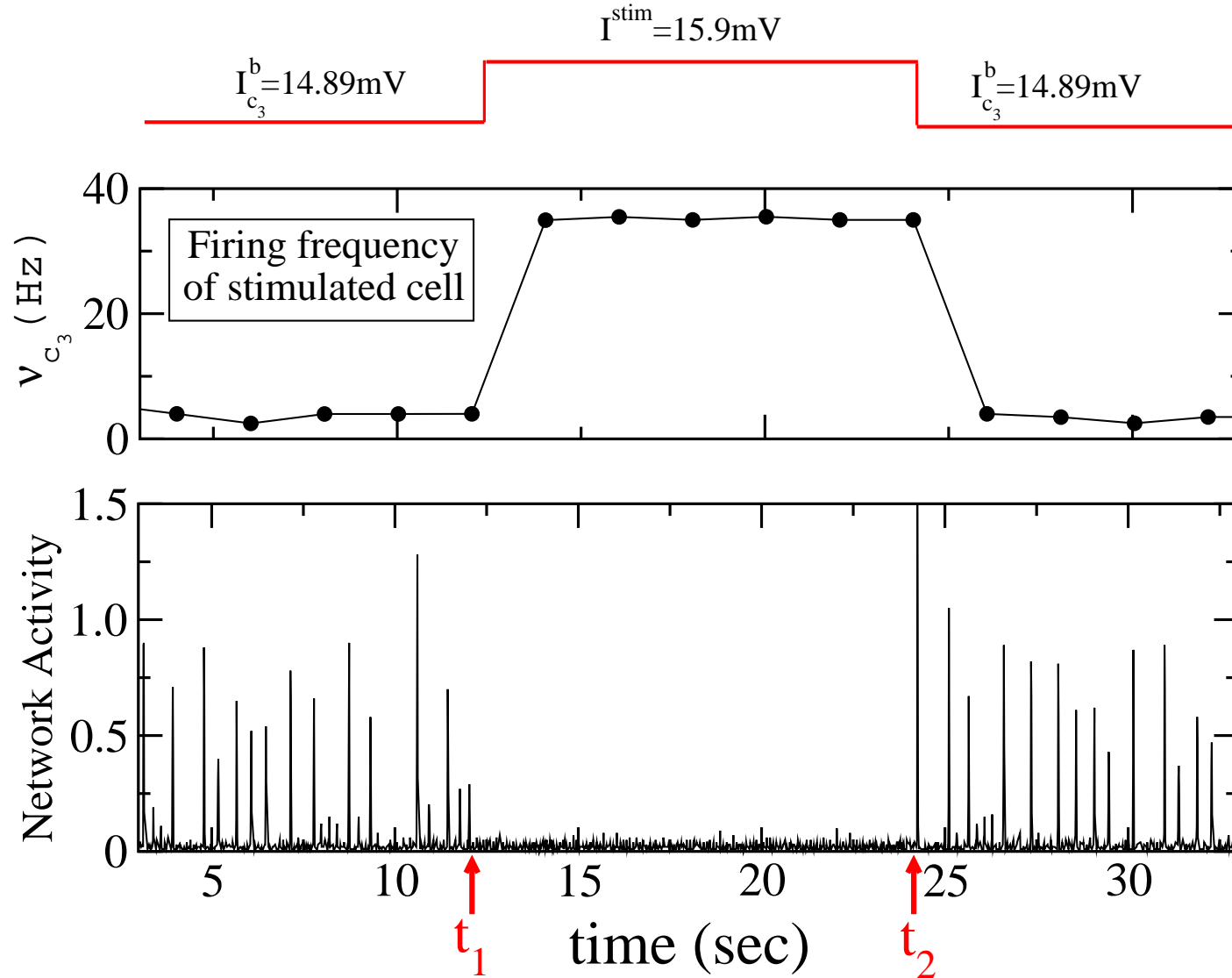


Ge et al, Nature (2005); Doetsch and Hen, Curr Op Neurob. (2005)  
Karayannis et al, Frontiers neural circuits (2012)

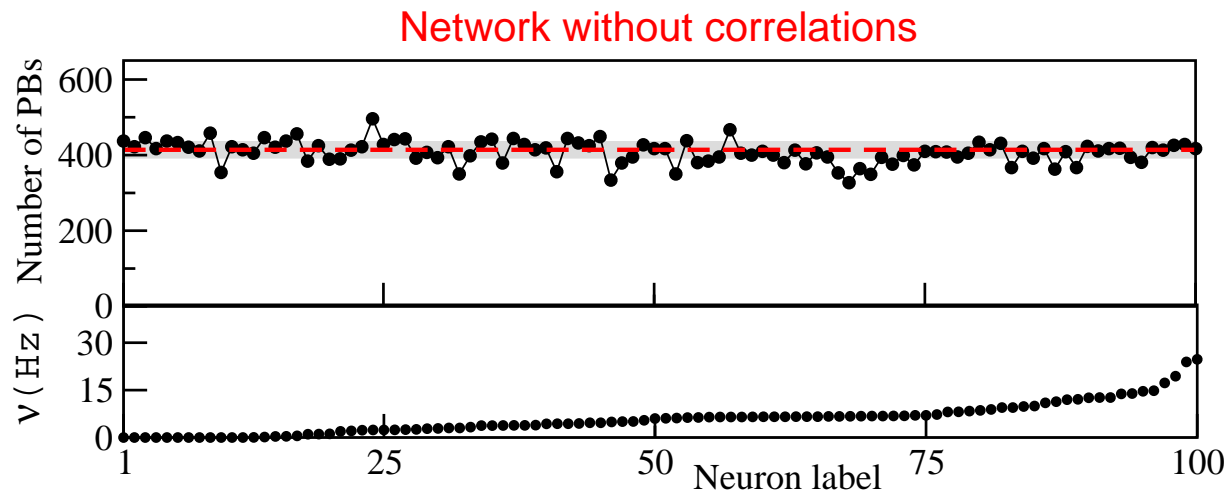
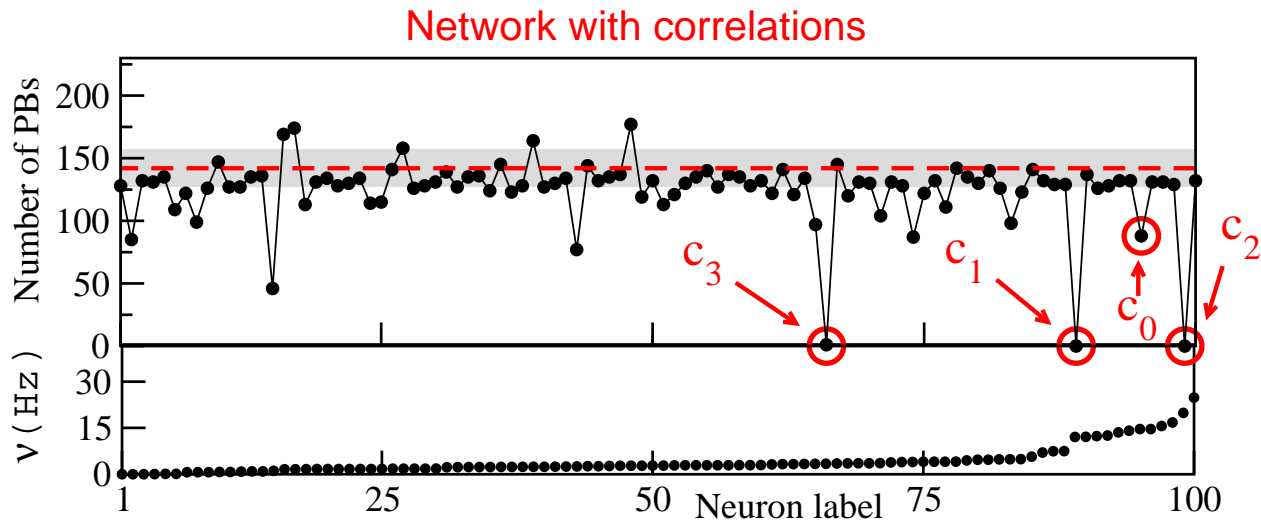
# Numerical Results

- Single Neuron Stimulation (SNS)
- Single Neuron Deletion (SND)

# Single neuron stimulation

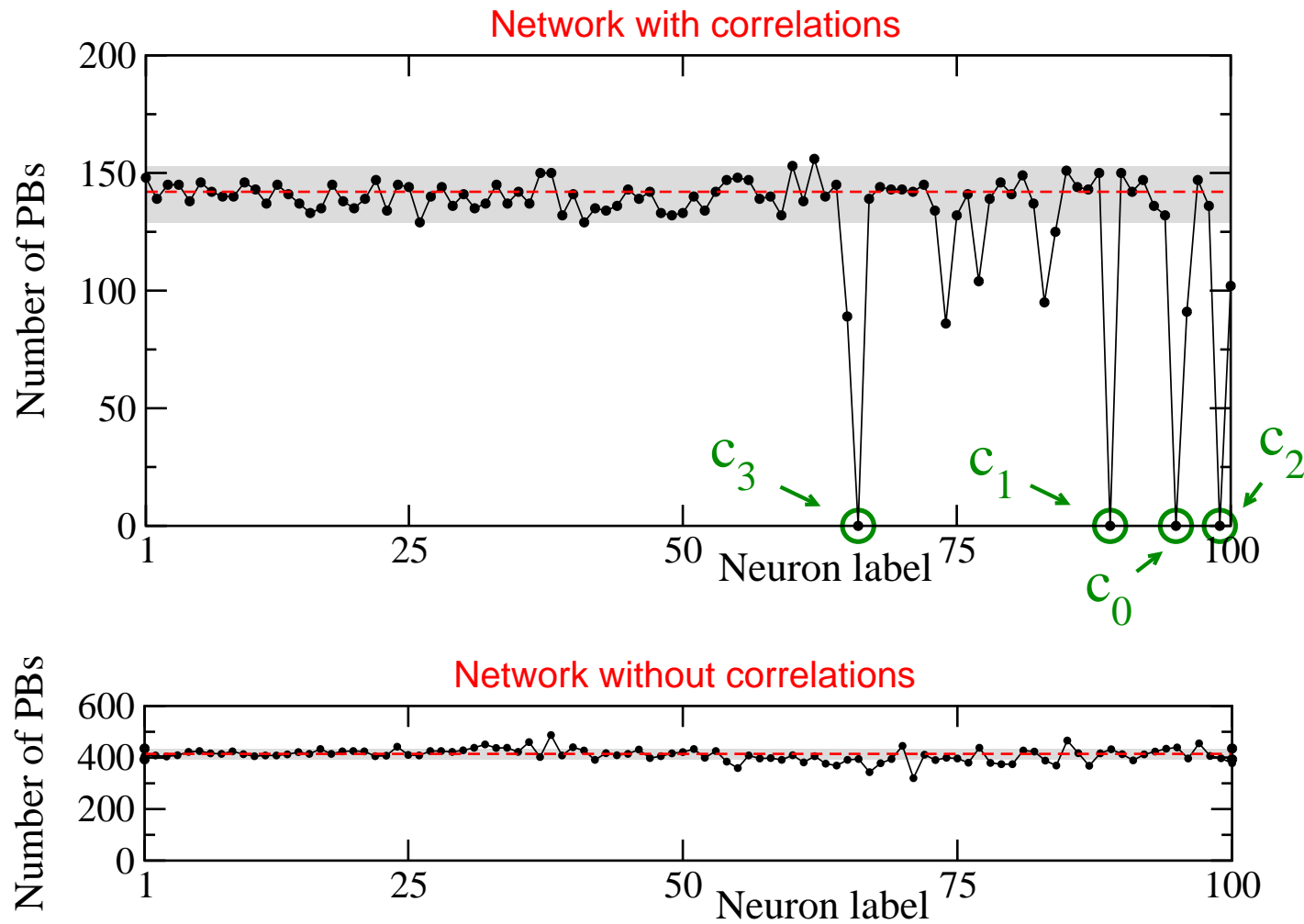


# Single neuron stimulation



Neurons are ordered according to their firing rate  $\nu$  under control conditions

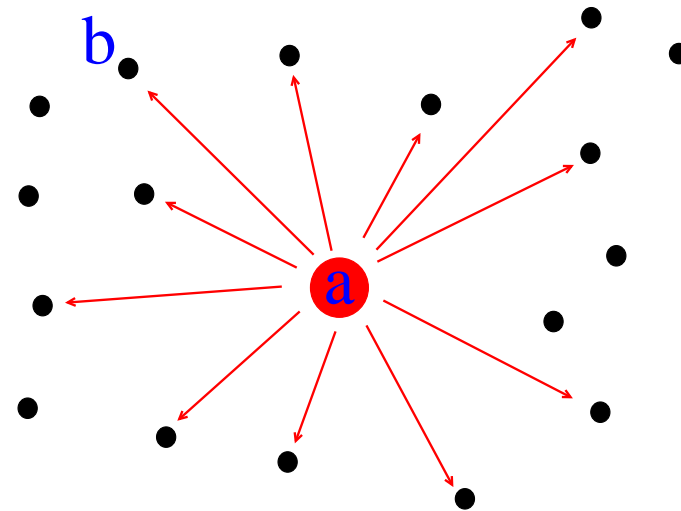
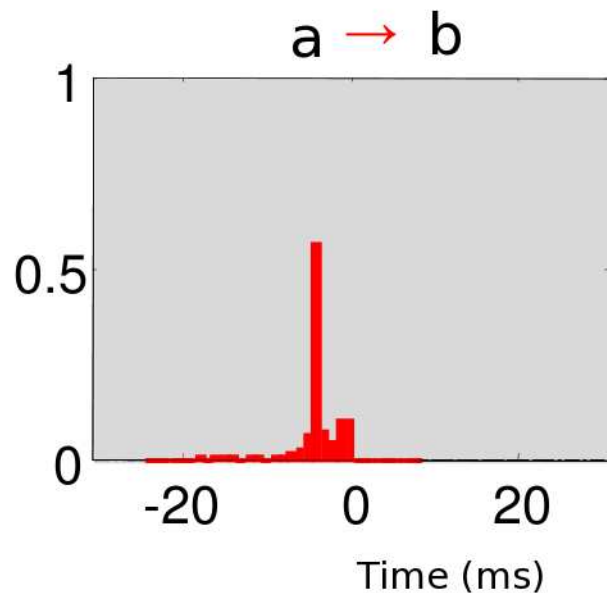
# Single neuron deletion





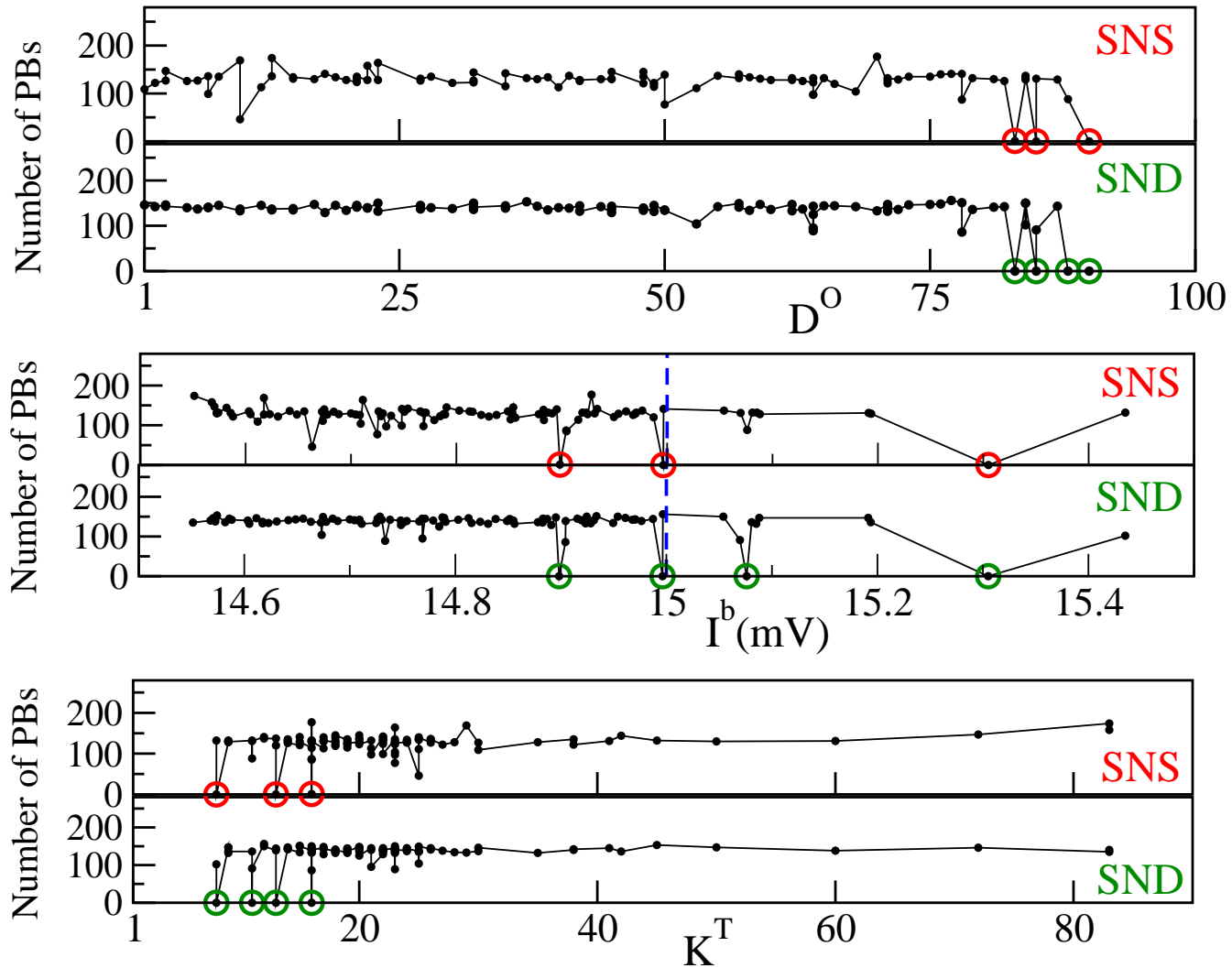
# Functional Connectivity analysis

Assignment of a functional degree to each cell of the network



$D_a^o$  = **functional out-degree** of cell  $a$  → number of cells activated after its firing

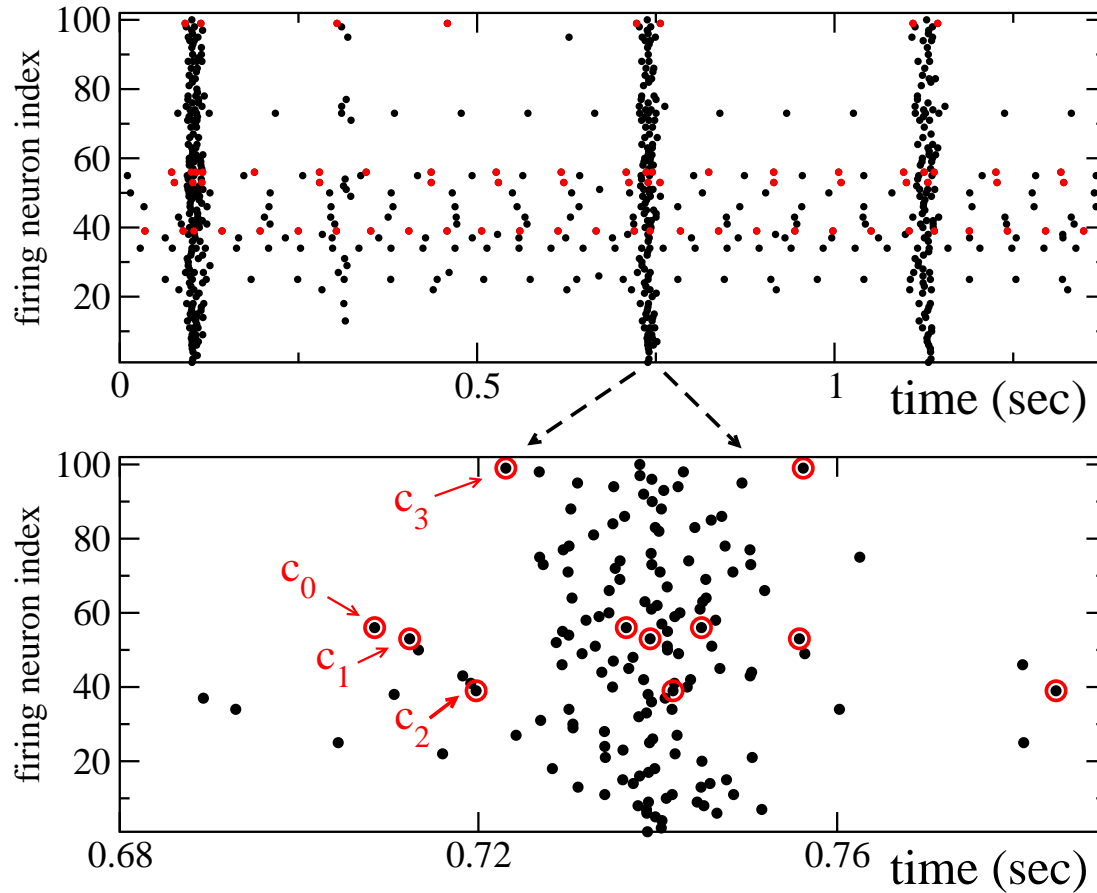
# Properties of critical neurons



# The functional clique

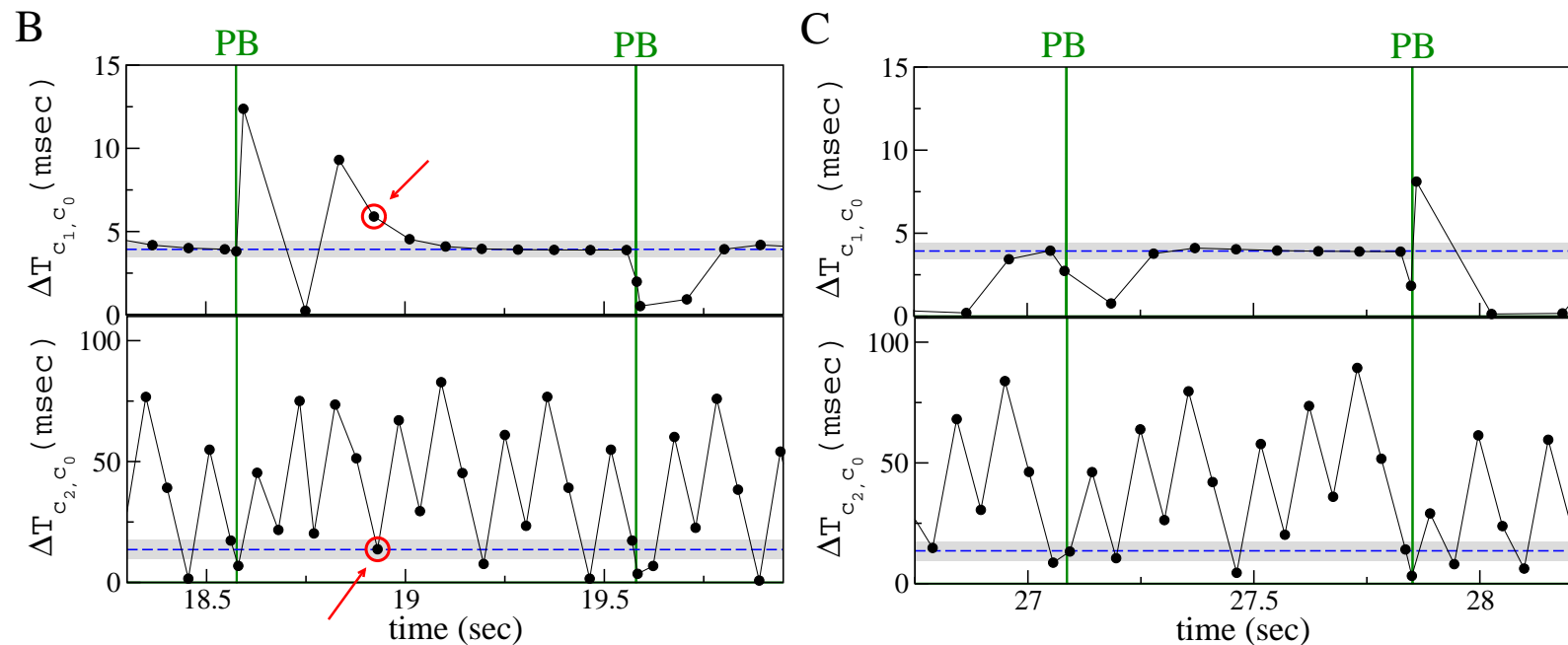
# Population Burst build up

Population bursts are anticipated by the sequence  $c_0 \rightarrow c_1 \rightarrow c_2 \rightarrow c_3$



$$\Delta T_{c_1, c_0} = 3.94 \pm 0.5 \text{ms} \quad \Delta T_{c_2, c_1} = 9.6 \pm 3.3 \text{ms} \quad \Delta T_{c_3, c_2} = 3.3 \pm 1.0 \text{ms}$$

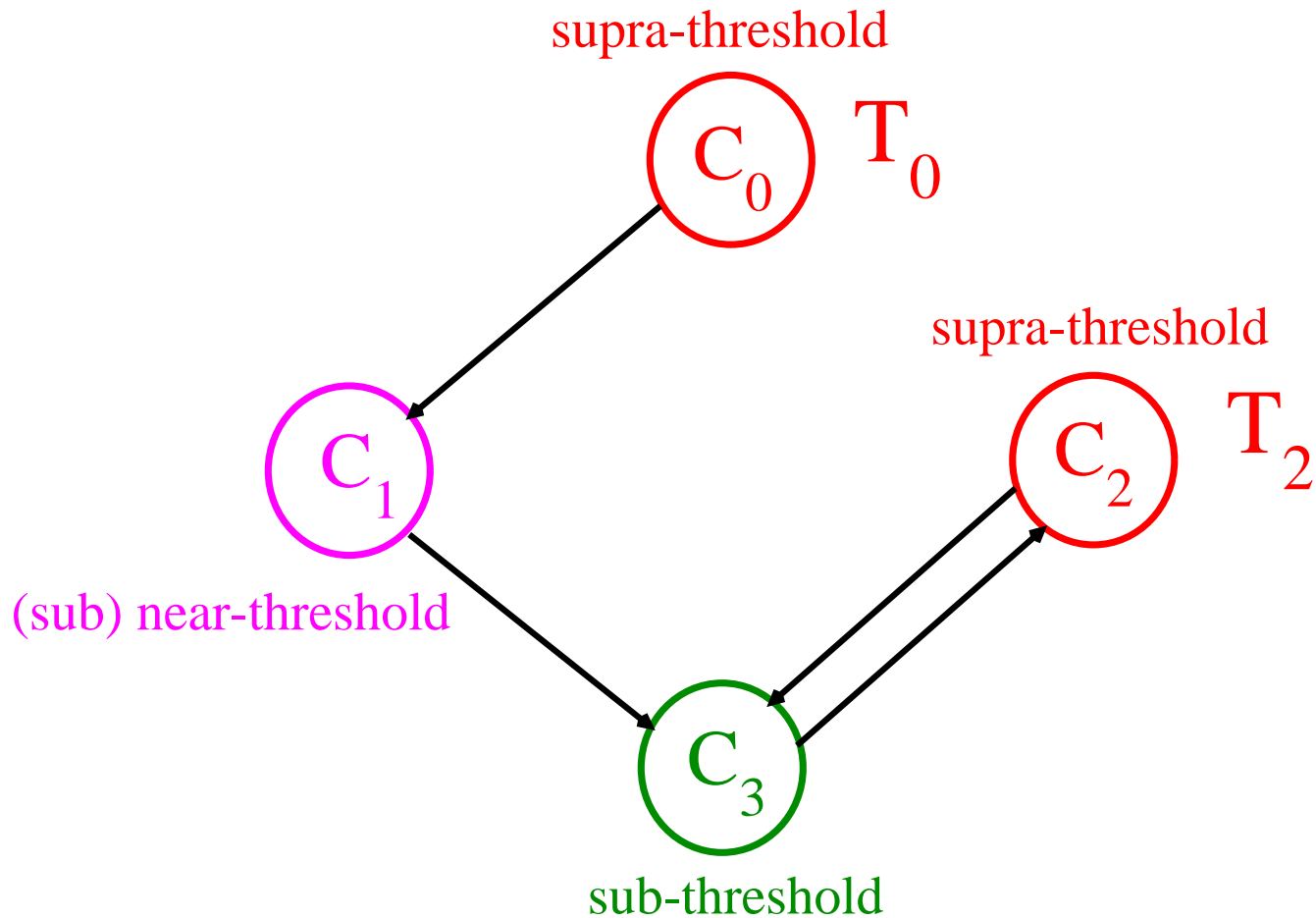
# Failures and successes in PB build-up







- $c_1$  fires almost always with a fixed time delay after  $c_0$ , apart just after PB emission
- $c_2$  fires quite independently with respect to  $c_1$  ( $c_2$  has his own evolution)

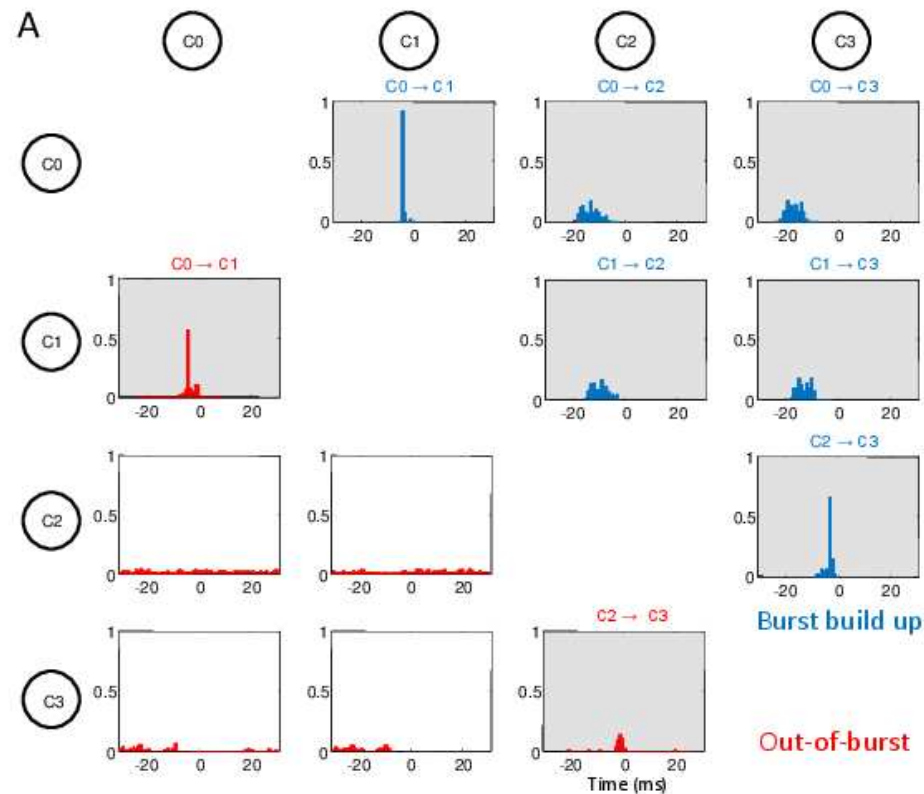
# The functional clique

## “Clocks” and “Followers”

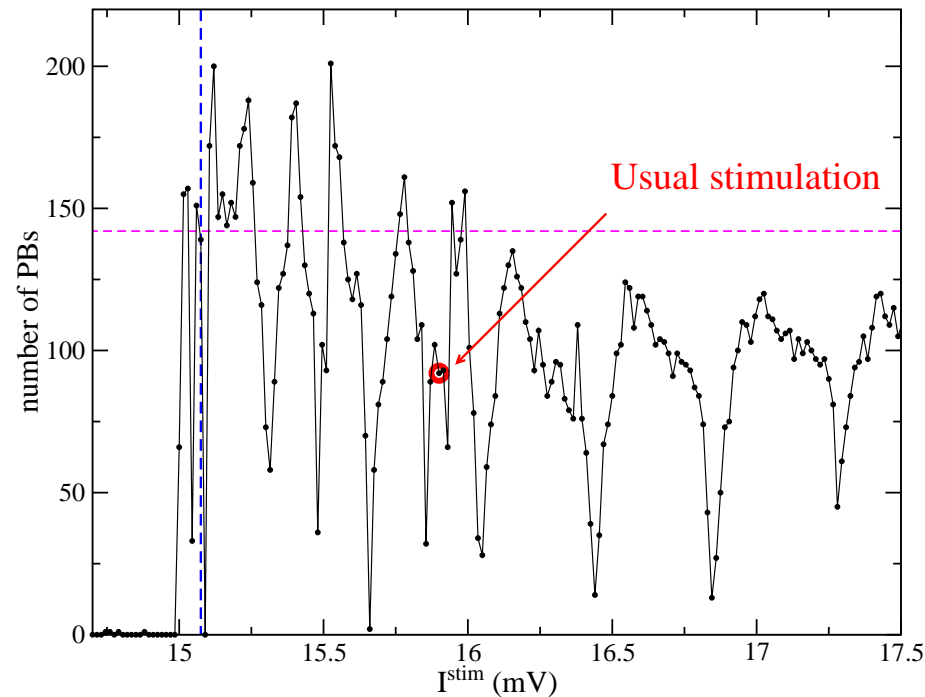


# The mechanism

-   $c_1$  follows the firing of  $c_0$  (also out of burst)
-   $c_3$  follows the firing of  $c_2$  (also out of burst)
-  No structural connections among the two “clocks”  $c_0$  and  $c_2$
-  Only when the two “clocks” fire unsupervised in the correct order and in a precise time sequence the PB can occur ( $\Delta T_{c_2 c_0} \simeq 15ms$ )



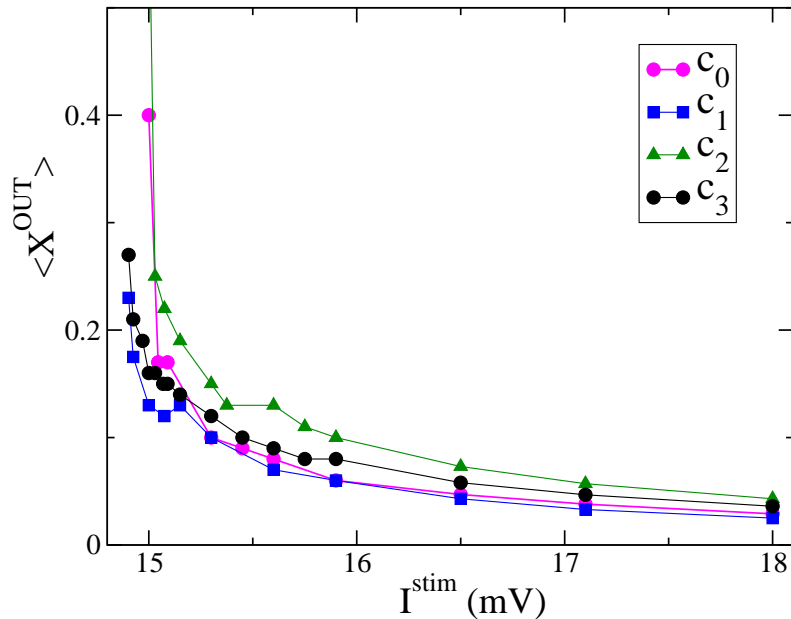
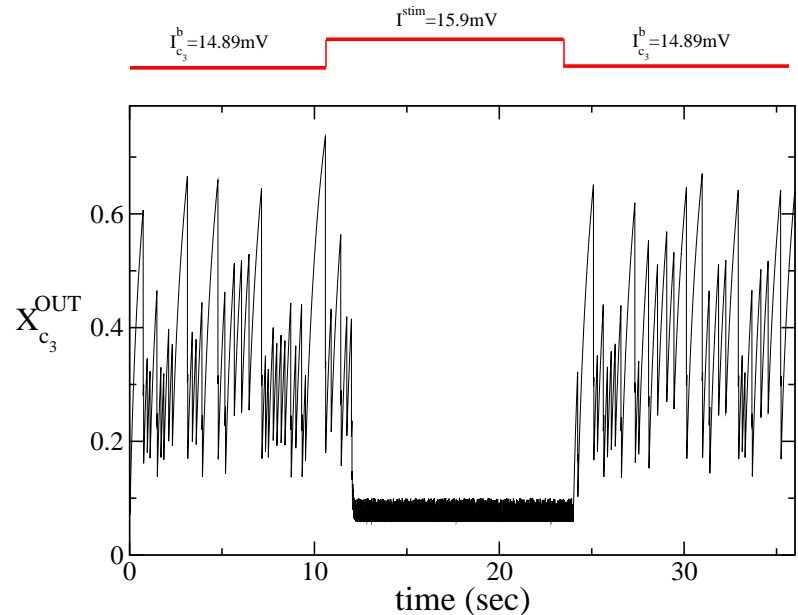
# Stimulating the clique leader $c_0$



- Minima in the PB activity (antiresonances) correspond to mode locking
  - The periods of the two "clocks"  $c_0, c_2$  are related  $mT_0 = nT_2$
  - The possibility that they fire with the required delay by chance goes almost to zero
- The number of PBs decreases with  $I^{stim}$



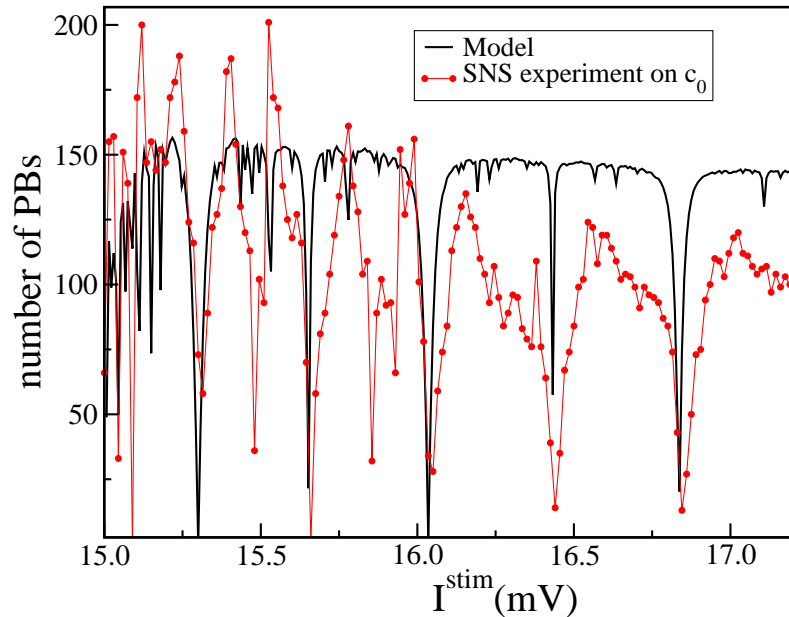
# The role of plasticity



$$X_i^{OUT} = \frac{1}{K_i^O} \sum_{k \neq i} \epsilon_{ki} X_{ki}$$

The afferent synapses are depressed by increasing the stimulation and this explains the decrease in the number of PBs with  $I^{stim}$ , but this does not explain antiresonances

# A simple model



We have developed a simple model to reproduce the stimulation experiment on  $c_0$

- $c_0$  and  $c_2$  are two independent LIF neurons suprathreshold
- The emission of a PB is probabilistic process controlled by the synaptic depression,  $\mathcal{P}(I^{stim}) \propto \langle X_{c_0}^{OUT} \rangle$

A PB is emitted whenever the following 2 conditions are fulfilled

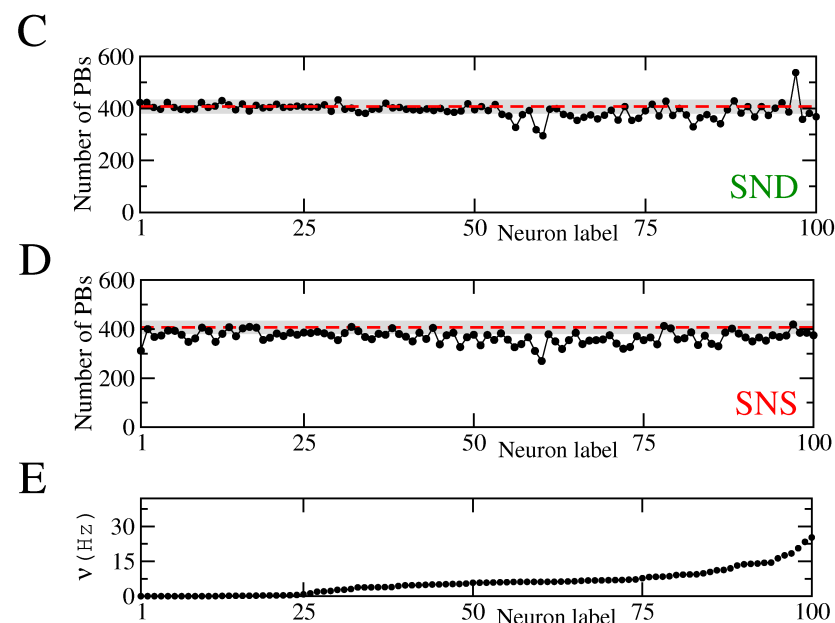
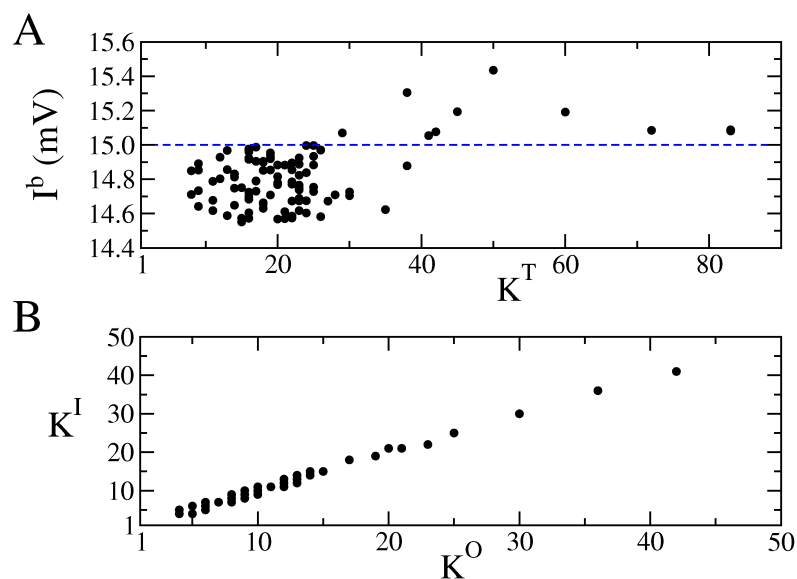
- $c_2$  fires after  $c_0$  with a time delay  $\Delta T_{c_2, c_0}$
- an extracted random number  $r < \mathcal{P}(I^{stim})$

This extremely simplified model reproduces more than 70% of the antiresonances

# Other correlation setups

- **setup T1:** positive correlation between the in-degree and out-degree of each neuron;
- **setup T2:** negative correlation between the intrinsic neuronal excitability and the total connectivity (in-degree plus out-degree);
- **setup T3:** positive correlation between the intrinsic neuronal excitability and the total connectivity (in-degree plus out-degree).

Correlated networks with all possible combinations of the setups T1-T3 have been examined, **only the networks with correlations T1 + T2 show sensitivity to SNS and SND**



# Conclusions



- We considered a **developmentally regulated** neural network
- We have found that :
  - single neuron stimulation/deletion of a few critical neurons is able to strongly impact the bursting activity
  - critical neurons are **functional hubs**
  - critical neurons are arranged in a **clique** controlling the neuronal activity
    - the clique is composed by “clocks” and “followers”
    - their ordered activations is required for the generation of a burst
    - the activation of the “clocks” is not mediated by structural connections
  - the synaptic resources regulate the interburst period, but they are not the key element for PB occurrence

We have verified the validity of our findings for other 6 realizations of the neural network

S.Luccioli, E. Ben-Jacob, A. Barzilai, P. Bonifazi, AT,  
PLOS Computational Biology (2014)

- The gradient of the excitability can be explained by assuming a less depolarizing action of GABAergic transmission on mature cells with respect to younger ones ?
- Some of our findings can be experimentally tested ?  
[Antiresonances — Sequential Activation of Neurons](#)
- What will change with scale-free topologies ?
- These results can be of interest for network controllability  
[Liu, Slotine, Barabasi, Nature \(2011\)](#); [Menichetti, Dall'Asta Bianconi, PRL \(2014\)](#)

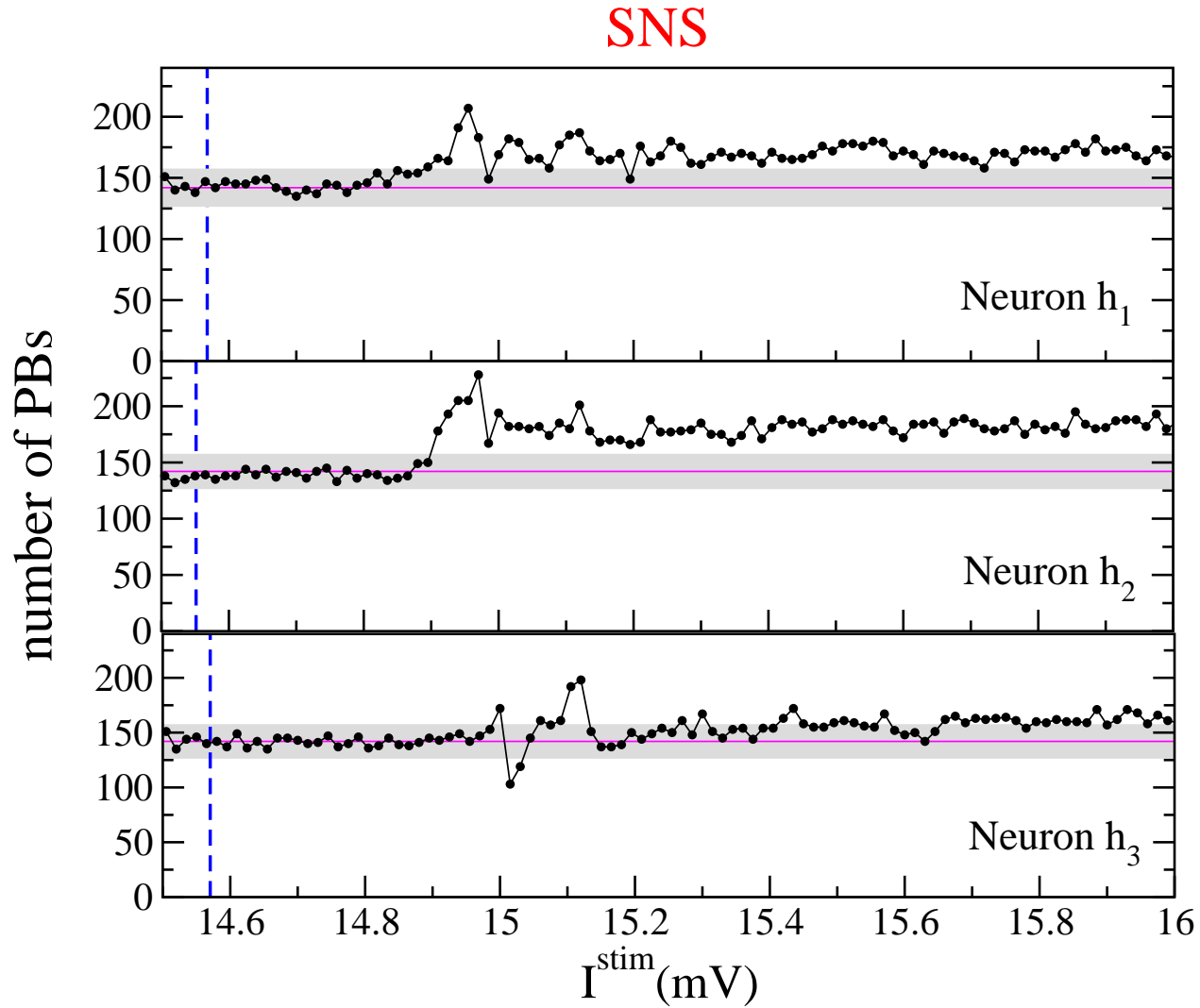
# Correlations are fundamental



Menichetti, Dell'Asta, Bianconi, Phys. Rev. Lett 113 (2014) 078701

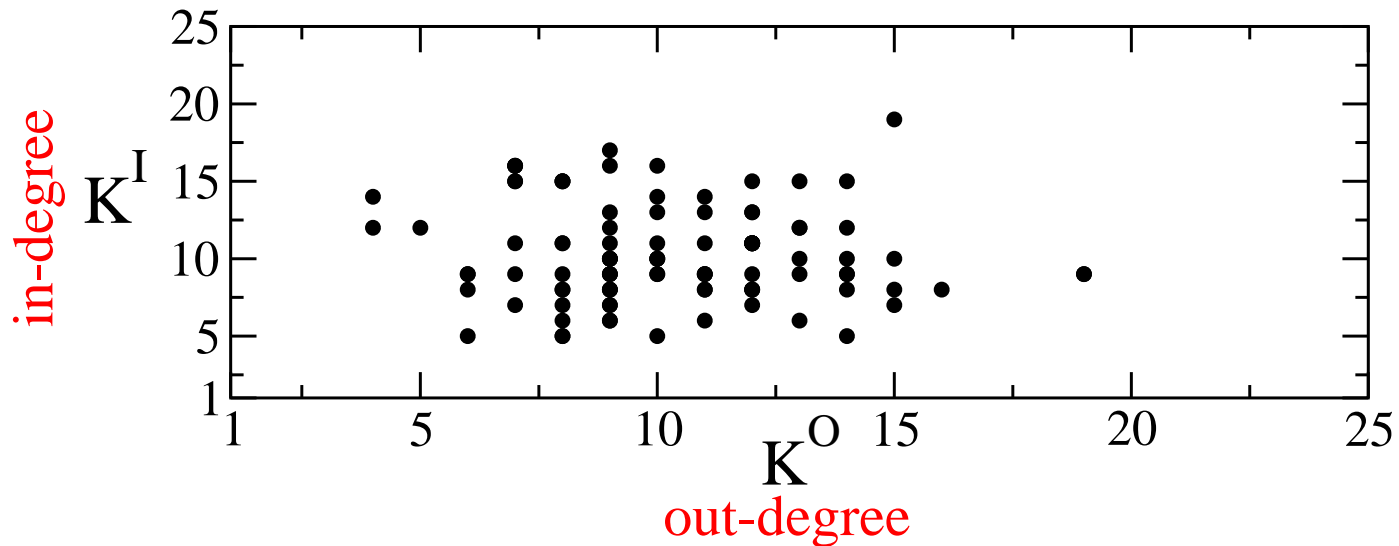
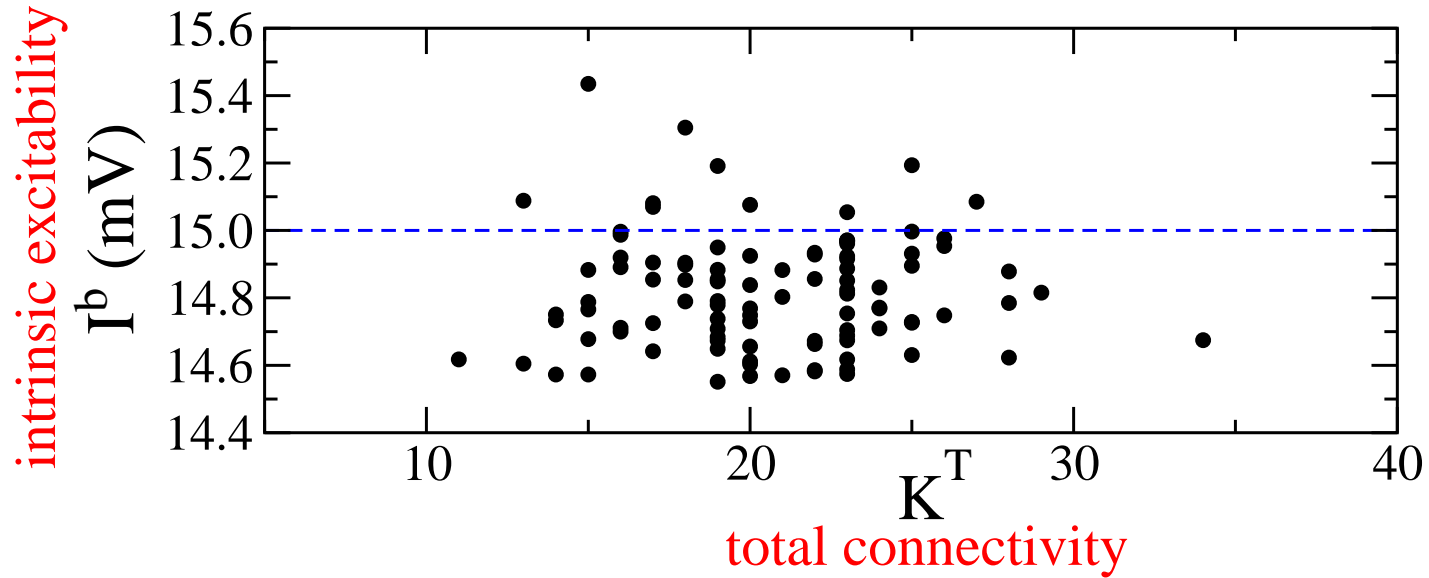
- Driver nodes  $N_D$  can bring the network to any desired dynamical state if a external signal is applied to them;
- In an **uncorrelated** directed random network  $N_D \equiv 0$  if  $K^{IN}, K^{OUT} > 2$
- Structural and Dynamical Correlations can change all the story
- The studied model is linear  $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$

# Role of structural hubs



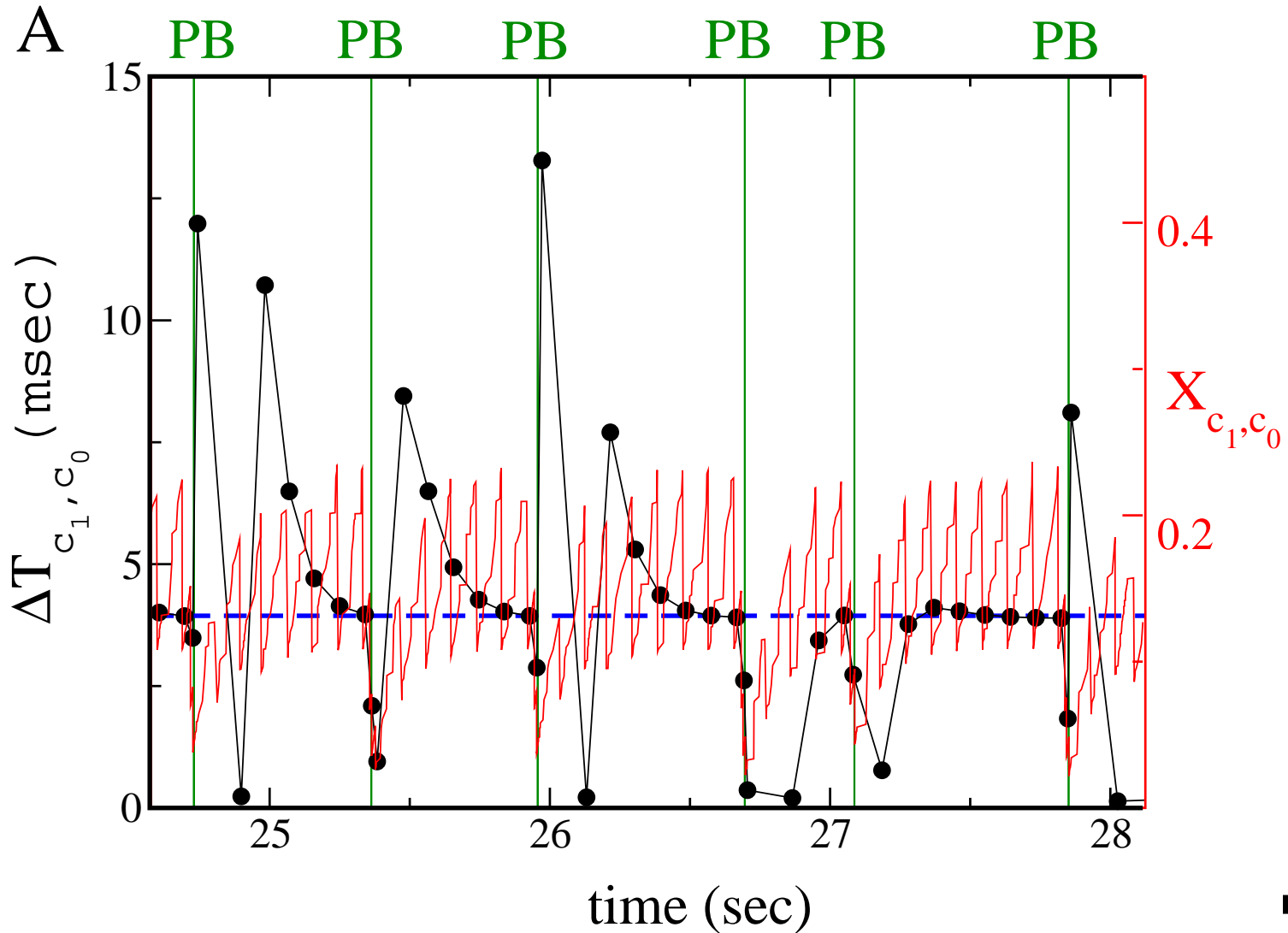
Three neurons have  $K^T > 60$  in the network

# Network without correlations

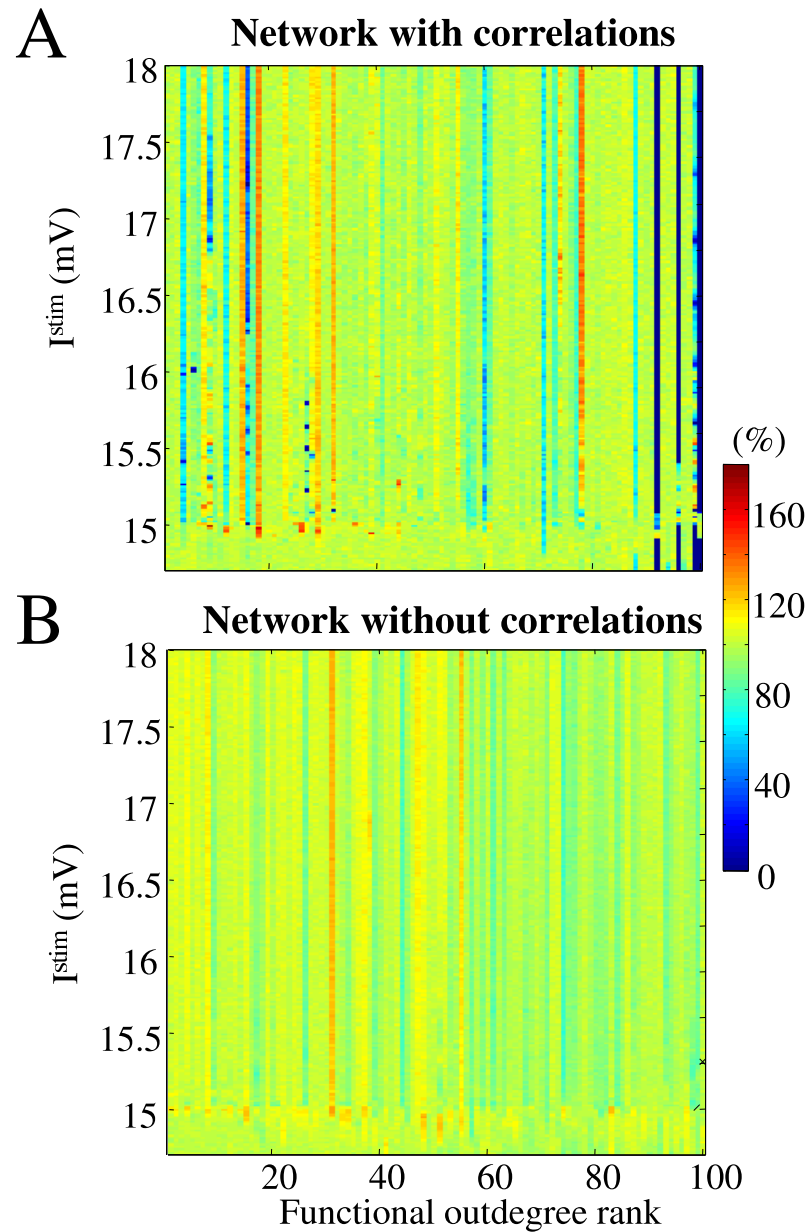




# Synaptic strength and firing time delay



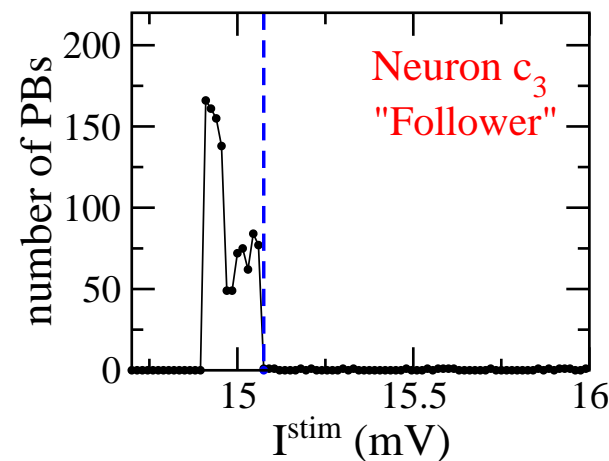
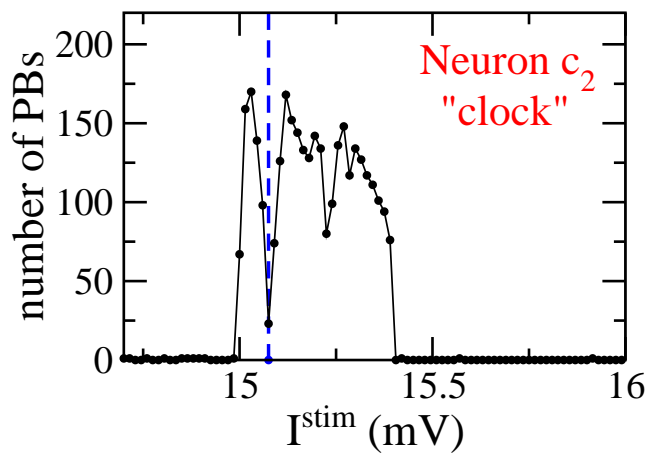
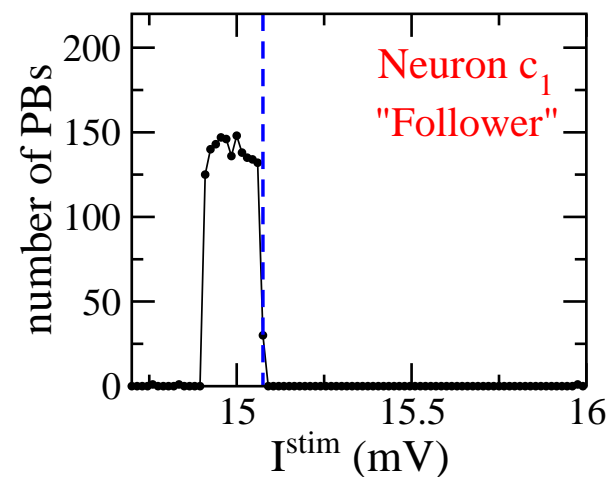
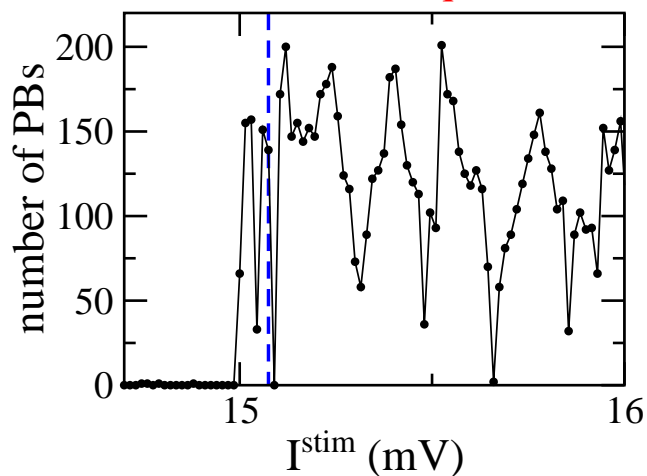
# Extensive single neuron stimulation



# SNS of the critical neurons

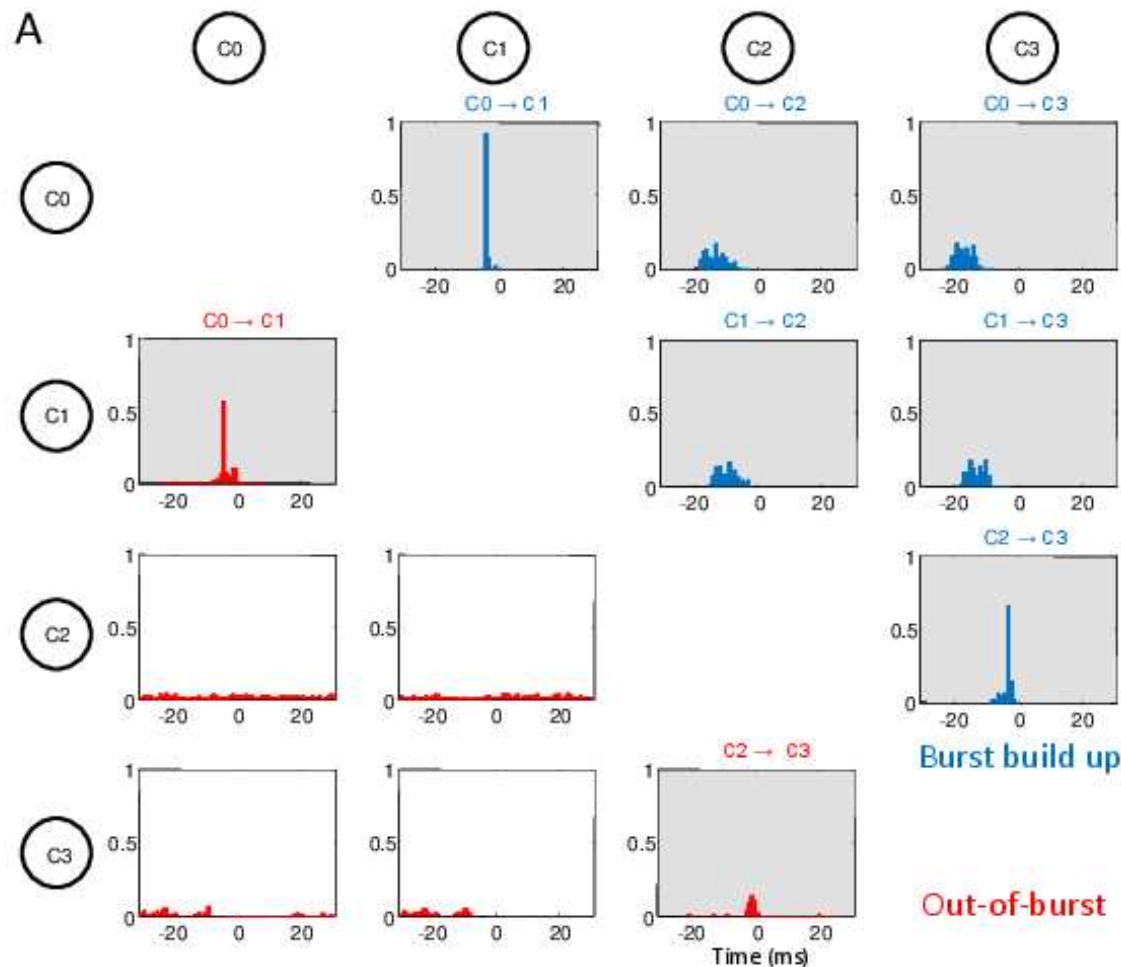
## Windows of bursting activity

Neuron  $c_0$   
"clock" and "clique leader"

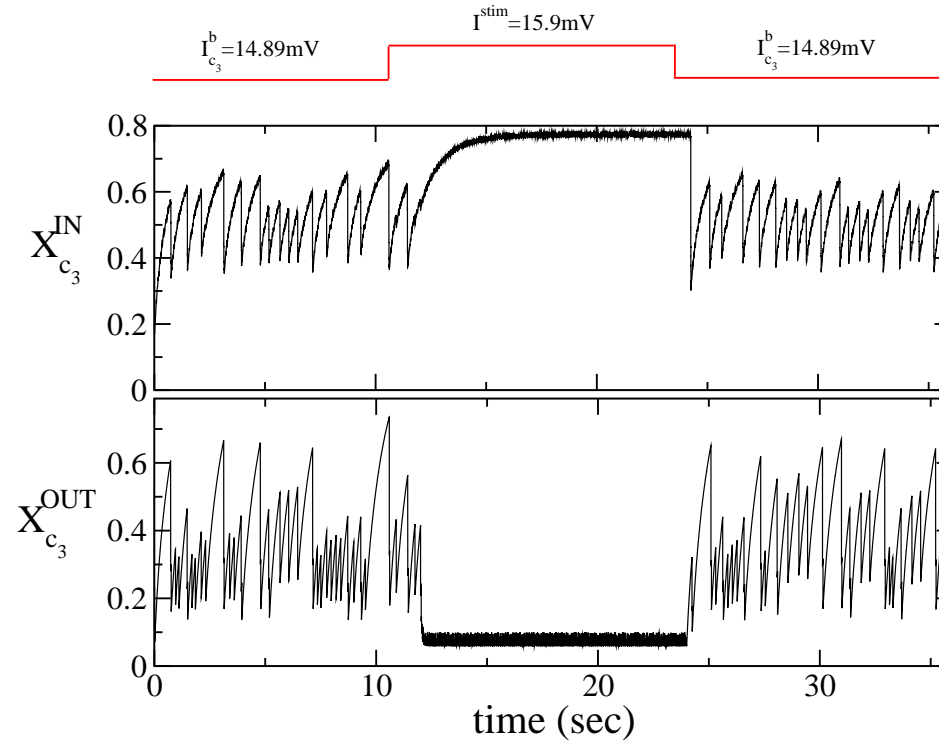


# The functional clique - 3

Cross-correlation analysis of the **out-of-burst** and of the **PB build up** firing times



# The role of plasticity



$$X_i^{OUT} = \frac{1}{K_i^O} \sum_{k \neq i} \epsilon_{ki} X_{ki} \quad ,$$

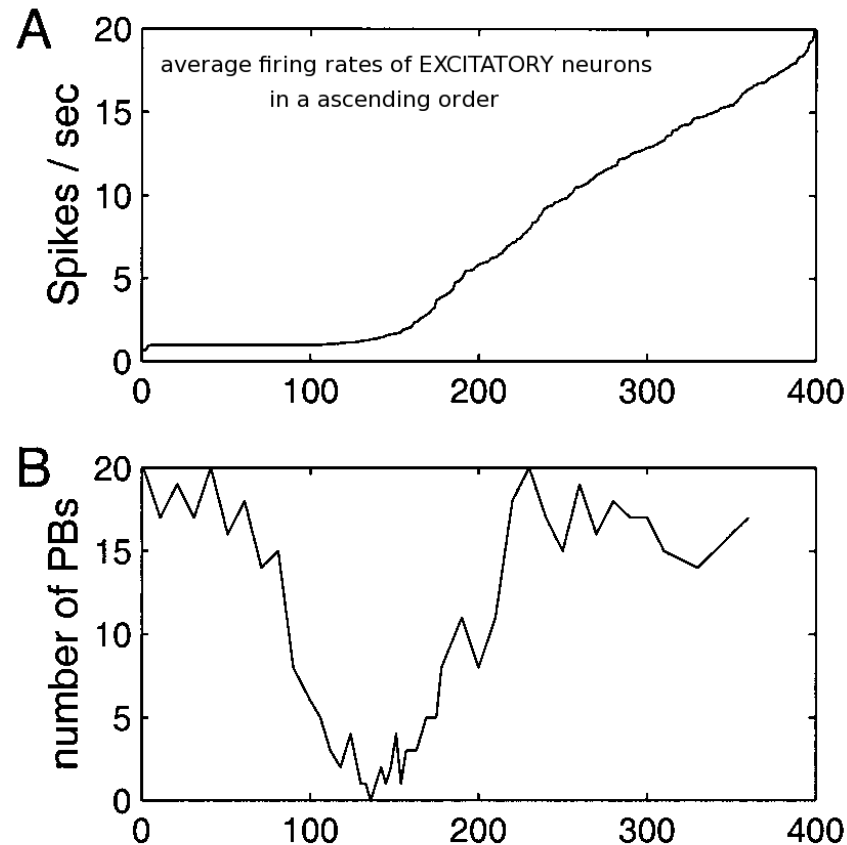
$$X_i^{IN} = \frac{1}{K_i^I} \sum_{j \neq i} \epsilon_{ij} X_{ij}$$

# Comparison with TUM experiment



$$N_{exc}=400, N_{inh}=100$$

Groups of 30 neurons taken out of the network (TUM, J. Neurosc. 2000)



Collective Effect

- Introduction of binary time series with one millisecond time resolution, where ones (zeros) marked the occurrence (absence) of the action potentials.
- Given the binary time series of two neurons  $\{a_t\}, \{b_t\}$ , the cross correlation was:

$$C_{ab}(\tau) = \frac{\sum_{t=\tau}^{T-\tau} a_{t+\tau} b_t}{\min(\sum_{i=1}^T a_i, \sum_{k=1}^T b_k)}$$

where  $T$  was their total duration.

- Whenever  $C_{ab}(\tau)$  presented a maximum at some finite time value  $\tau_{max}$  a functional connection was assigned between the two neurons: for  $\tau_{max} < 0$  ( $\tau_{max} > 0$ ) directed from  $a$  to  $b$  (from  $b$  to  $a$ ).
- A directed functional connection cannot be defined for an uniform cross-correlation corresponding to uncorrelated neurons or for synchronous firing of the two neurons associated to a Gaussian  $C_{ab}(\tau)$  centered at zero.
- To exclude the possibility that the cross correlation could be described by a Gaussian with zero mean or by a uniform distribution we employed both the Student's t-test and the Kolmogorov-Smirnov test with a level of confidence of 5%.
- The functional out-degree  $D_i^O$  (in-degree  $D_i^I$ ) of a neuron  $i$  corresponds to the number of neurons which were reliably activated after (before) its firing.