LFFs in Vertical Cavity Lasers

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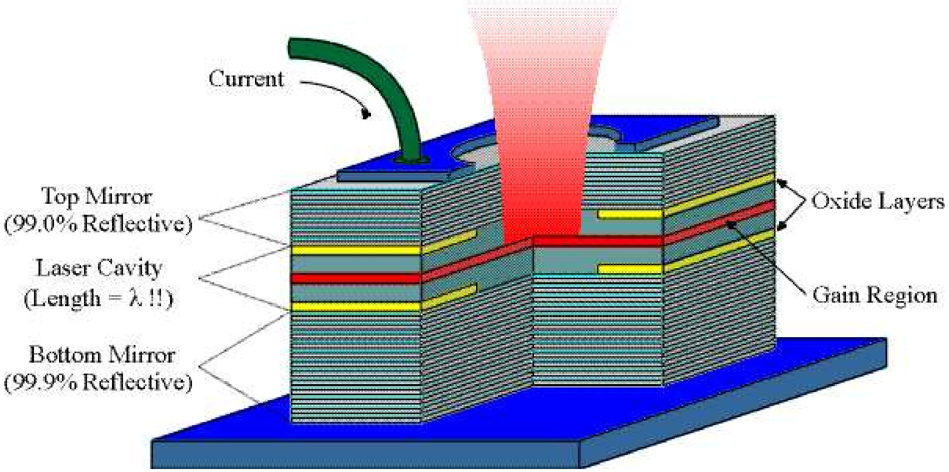

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- Comparison of numerical and experimental results
- Concluding remarks

VCSEL in brief

Single laser

- Semiconductor laser
- Wavelength: near IR (~ 800 nm) (optimal coupling with optical fibers)
- Single longitudinal mode, multiple transverse modes.
- Two linearly polarized emissions (symmetrical cavity).
- Good beam quality.

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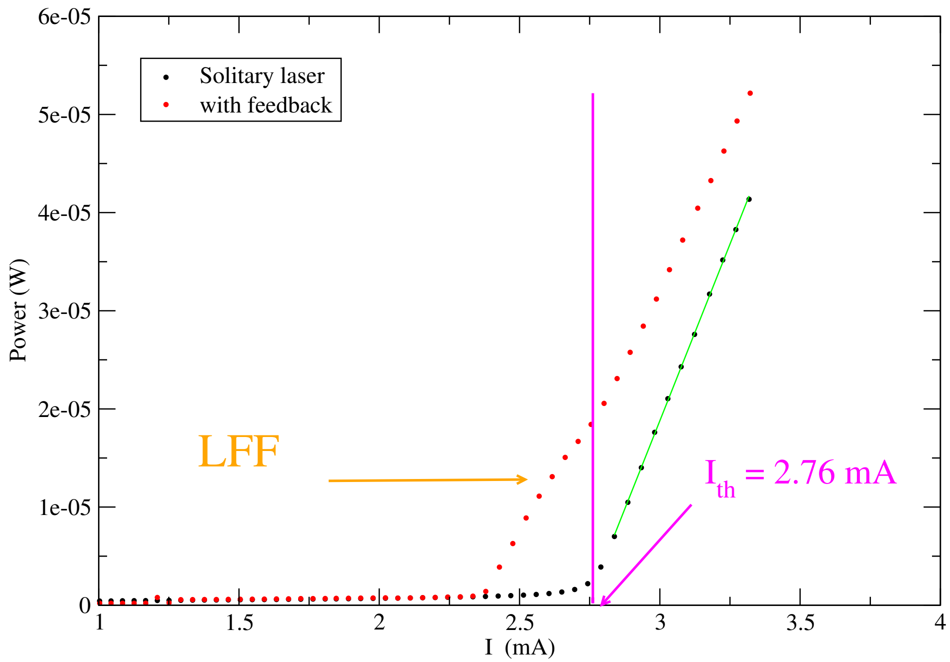
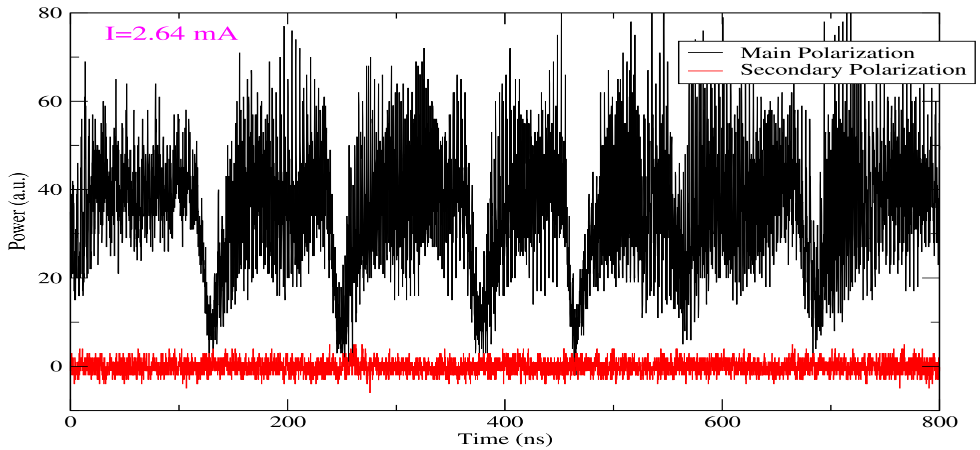
VCSEL with delayed optical feedback

- External cavity ~ 50 cm, round trip time $\tau = 3.63$ ns
- Polarization selective optical feedback

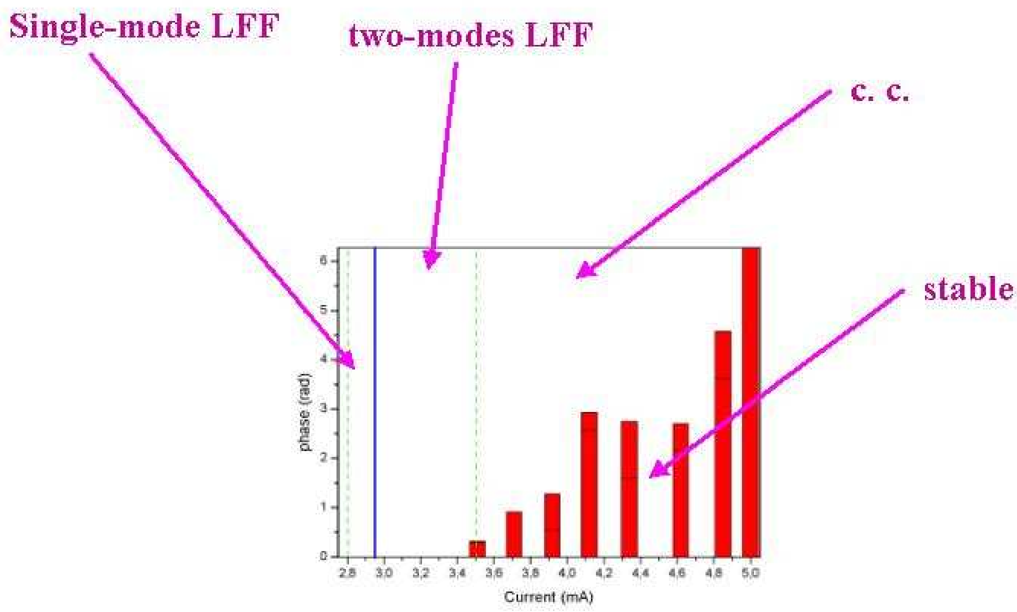
Dynamics of the VCSEL





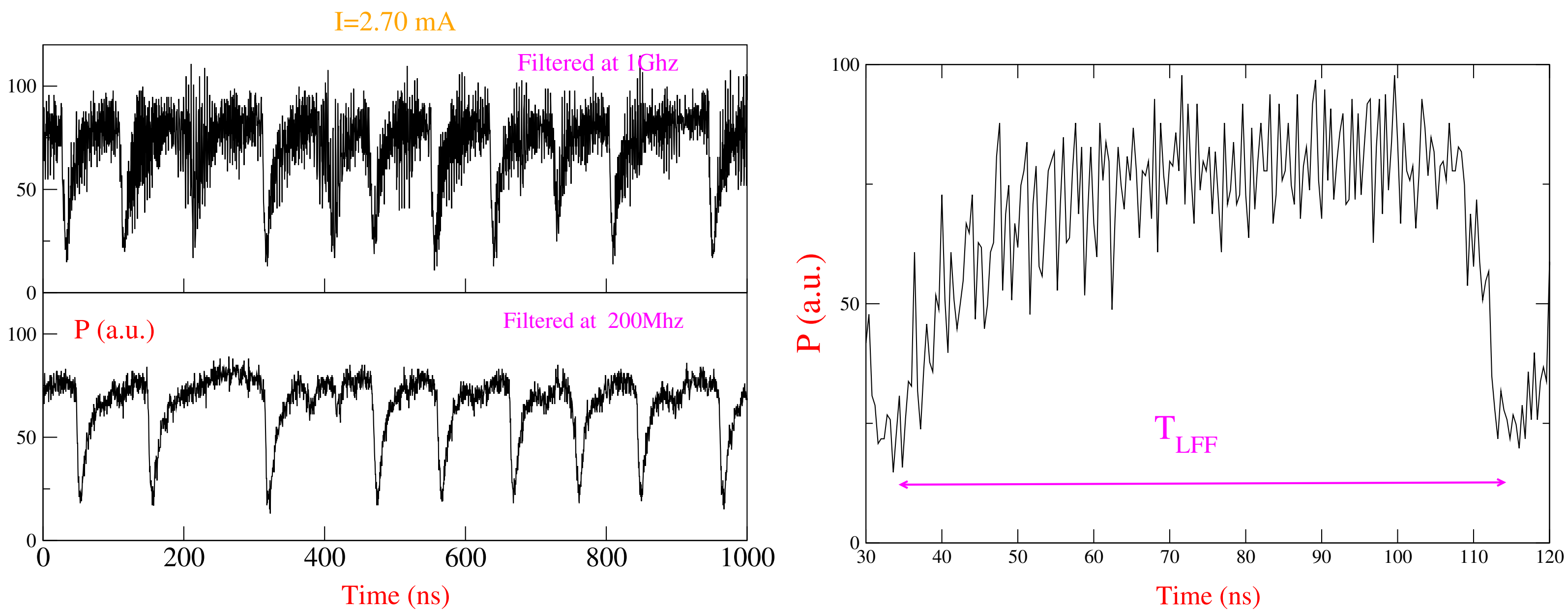
- for $I < 6$ mA the device emits linearly polarized light in the fundamental mode (the transverse modes are not active);
- for $I < I_{th}$ + feedback
  - the VCSEL is a single mode on the main polarization and its dynamics exhibits LFFs ;
  - the secondary polarization is absent.

VCSEL Phase Diagram



We will examine the single mode LFF regime, where the dynamical behaviour is not particularly influenced by the phase delay induced by the feedback.

LFF in a nutshell



- LFFs are feedback induced instabilities;
- the light is emitted in short pulses ~ 0.1 ns ;
- the filtered intensity grows to a almost constant value over a cycle of duration ~ 100 ns then drops to a much lower value;
- the LFF cycles can coexist with stable emission on a high-gain mode.

Lang-Kobayashi eqs

The dynamics of the VCSEL with delayed optical feedback is described for $c_0 = I/I_{th} < 1$ by the rate eqs.:

$$\begin{aligned} \tau_n \dot{N}(t) &= 1 + \eta(c_0 - 1) - N(t)(1 + |E(t)|^2) \\ \dot{E}(t) &= \left(\frac{1 + i\alpha}{2\tau_p}\right)(N(t) - 1)E(t) + \frac{k}{\tau_p} \mathrm{e}^{-\omega_0 \tau} E(t - \tau) + \sqrt{\frac{R_{sp}}{2}} \xi(t) \end{aligned}$$

where $E(t) = \rho(t) \exp i\phi(t)$ is the complex field, $N(t)$ the carrier density, and $\xi(t)$ a Gaussian noise term (spontaneous emission).

The parameters have been experimentally measured: $\alpha = 3.3 \pm 0.1$, $\tau_n = 0.37 \pm 0.02$ ns , $\tau_p = 12 \pm 1$ ps , $\eta = 5.8$, $\tau = 3.63$ ns , $R_{sp} = (2.3 \pm 0.5) \cdot 10^{-4}$ ps^{-1} .

k will be determined by comparison of numerical vs experimenal data.

S. Barland, P. Spinicelli, G. Giacomelli, F. Marin, IEEE J. Quantum Electronics (2005).

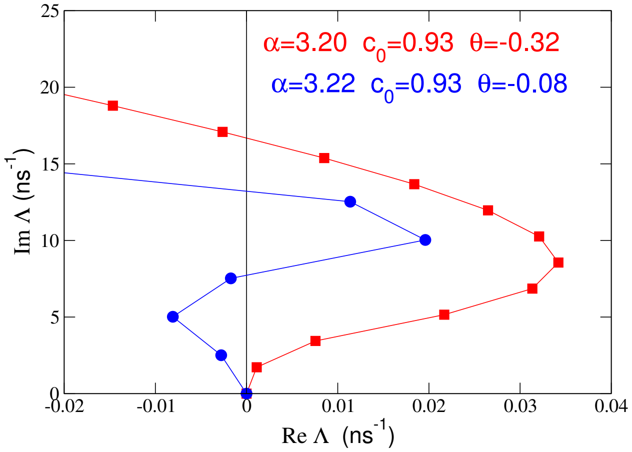
Stationary solutions

The deterministic LK eqs admit stationary solutions of the type:

$$E(t) = \rho_0 \exp(i\Omega t) \quad N(t) = N_0 = \text{const.}$$

The unstable solutions with a real positive eigenvalue are termed Antimodes, the other solutions can be stable or unstable depending on the parameters and are termed Modes.

The Modes can be destabilized by different mechanisms: e.g. Modulational or Turing instabilities.



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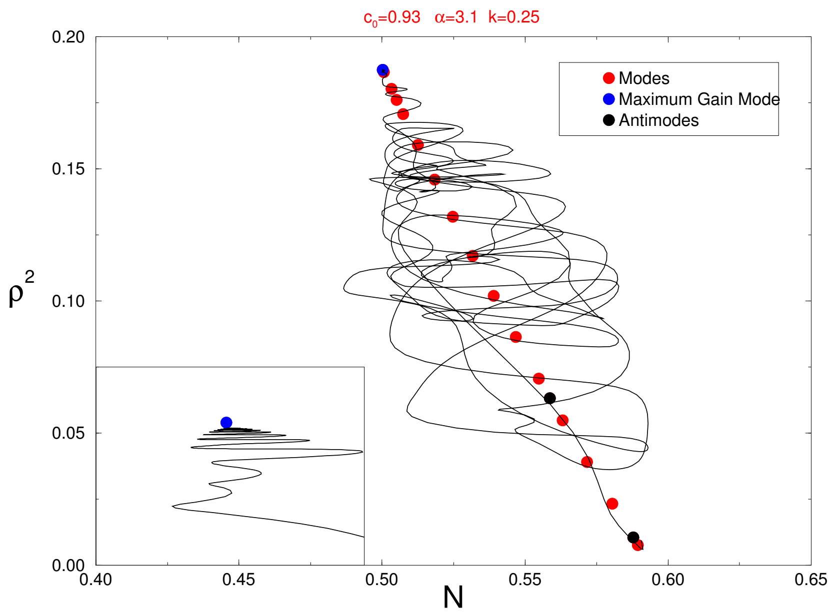


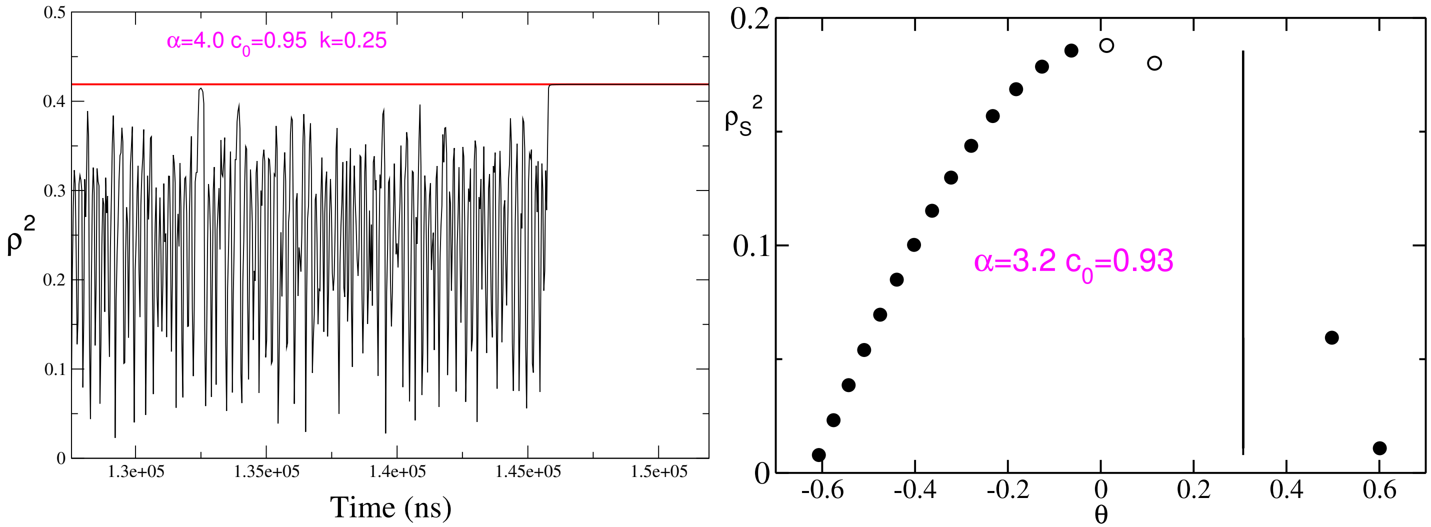
- Stable modes are more probable for small α or k -values.
- The maximum gain mode is stable and it coexists with the LFF dynamics.
- Two/three (maximum gain) modes can be stable at the same time.

S. Yanchuk & M. Wolfrum, WIAS preprint n. 962, Berlin (2004)

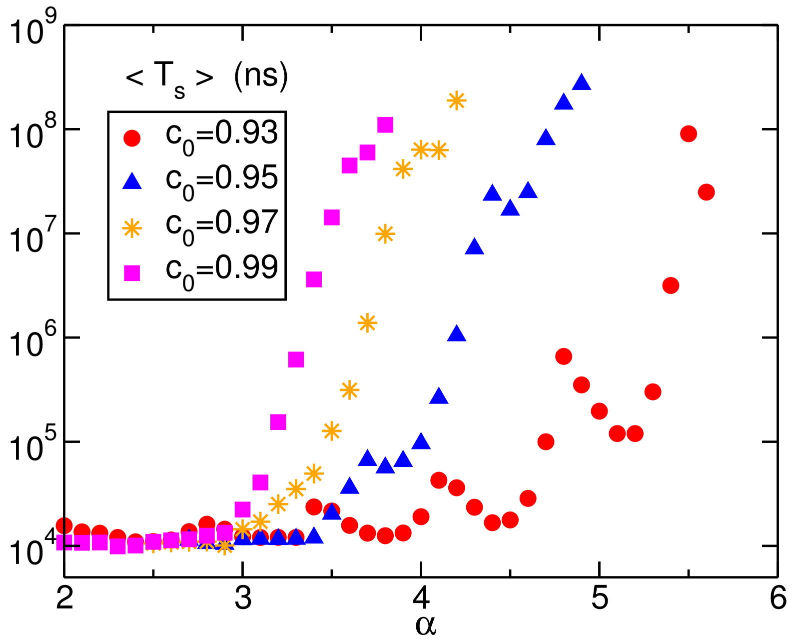
Deterministic dynamics

- The LFF dynamics is due to the chaotic itinerancy from one quasi-attractor to another, where the quasi-attractors are the ruins of a local chaotic attractor emerged via a period doubling bifurcations from the corresponding mode.
- On each local quasi-attractor the dynamics is chaotic but transient , the trajectory jumps from one quasi-attractor to another climbing towards the maximum gain mode .
- When the trajectory collides with an anti-mode the intensity drop and the dynamics restart again from low gain modes. (Sisyphus Cycles)

This is the deterministic explanation of LFFs (T. Sano, PRA 50 (1994) 2719)



LFF as a transient phenomenon

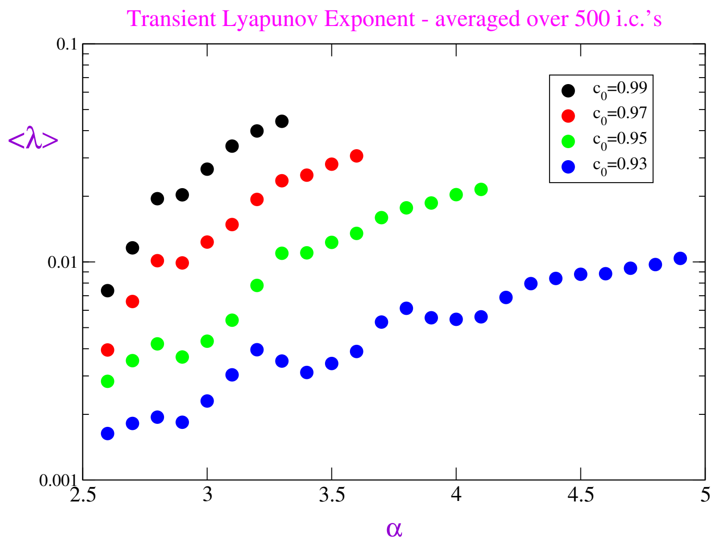


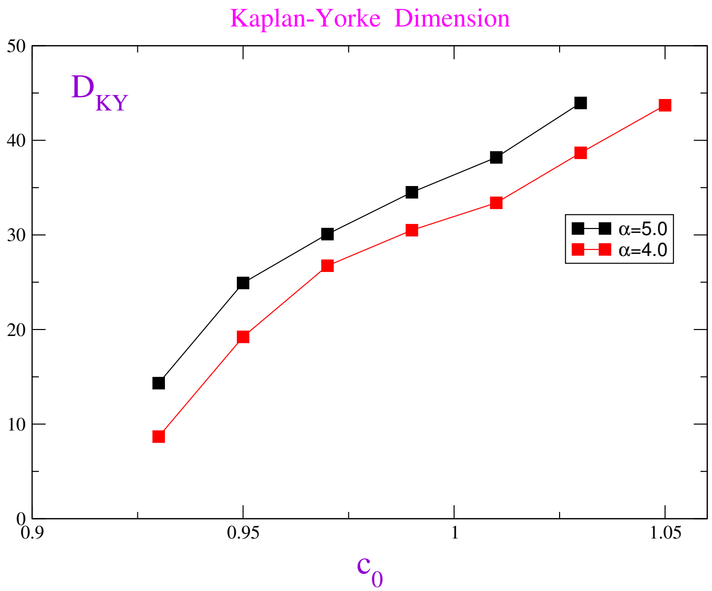
Below the threshold of the solitary laser ($c_0 < 1$) and for meaningful values of $\alpha = 3 - 5$ the LFFs are present only as a transient phenomenon.

The average transient time $< T_S >$ diverges for increasing c_0 and α .

Preliminar indications have been reported in T. Heil et al., Optics Let. 24 (1999) 1275

Lyapunov analysis



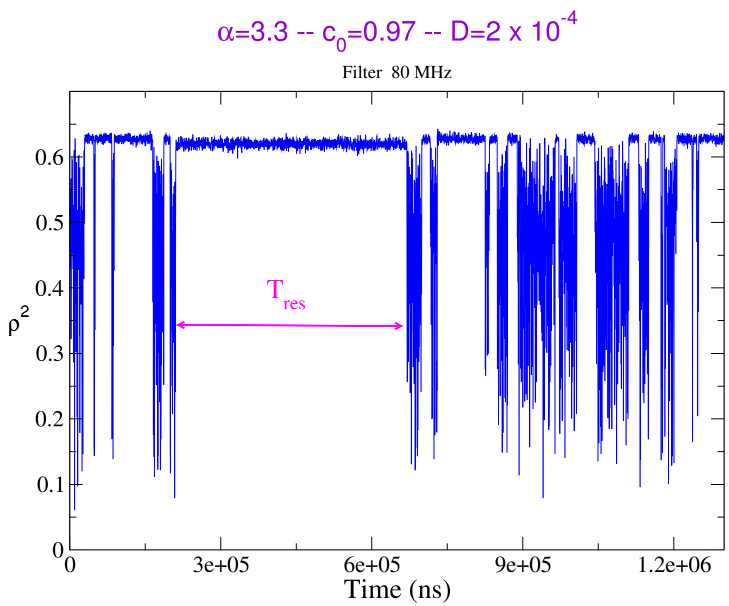


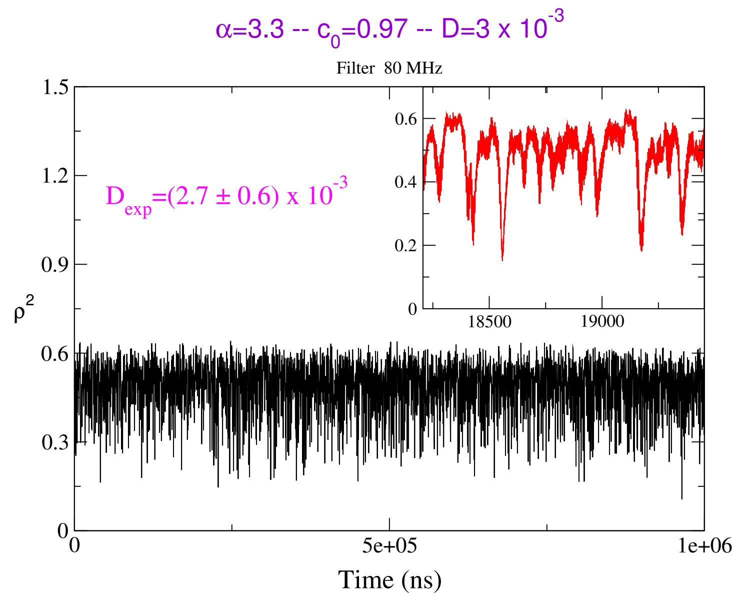
- The transient dynamics is chaotic and the maximal Lyapunov increases with c_0 and α .
- The number of active degrees of freedom (measured by D_{KY}) involved in the dynamics increases with c_0 and α .
- The system is not low dimensional.

Stochastic Dynamics I

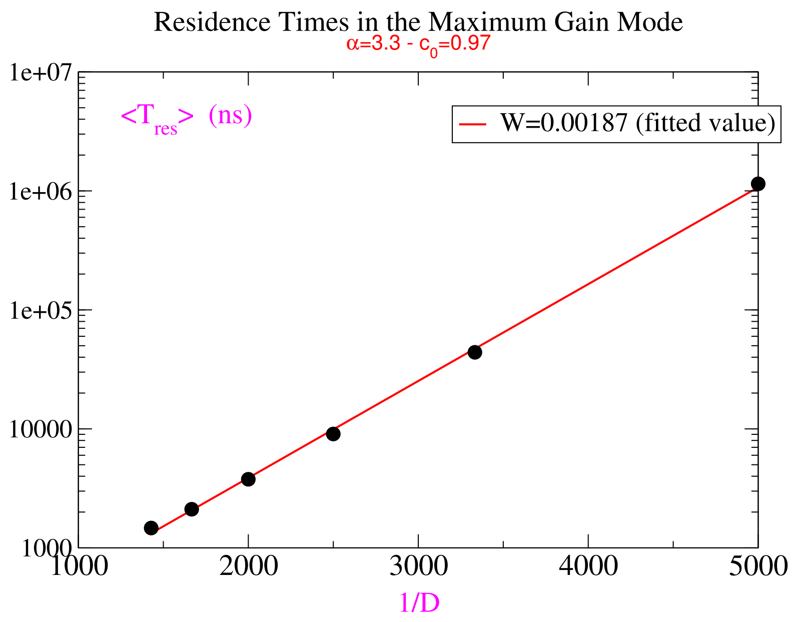
The presence of additive gaussian noise of variance D in the LK eqs. can destabilize the maximum gain mode leading

- for small noise to an intermittent behaviour ;
- for larger D values to a non transient LFF dynamics .





Stochastic Dynamics II



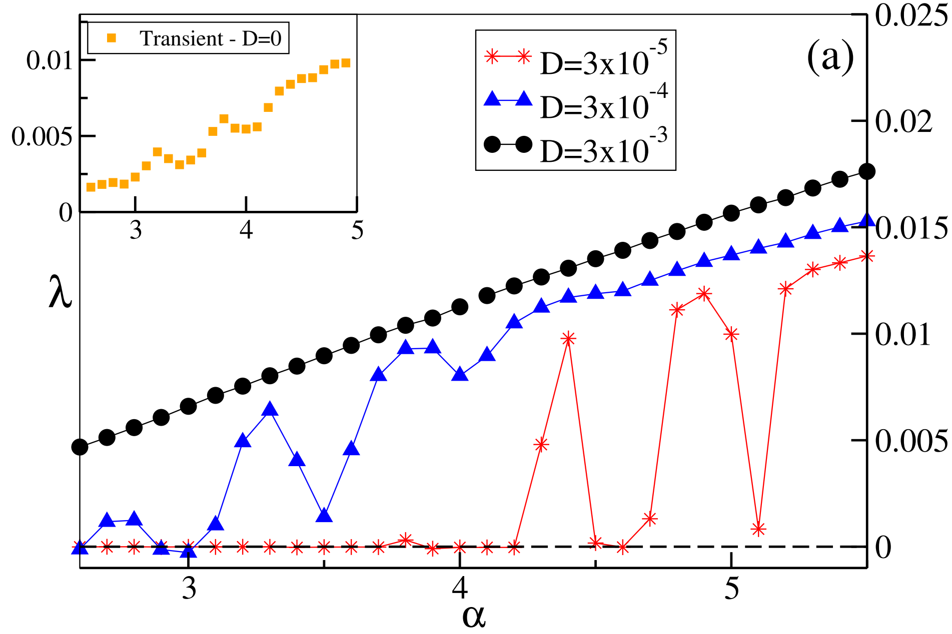
The intermittent dynamics can be seen as a stochastic escape process from the MGM induced by noise fluctuations

$$T_{res} \propto e^{W/D}$$

where the barrier height W for $\alpha = 3.3$ almost corresponds to the experimental value of the variance of the noise

$$D_{exp} = (2.7 \pm 0.6) \times 10^{-3}$$

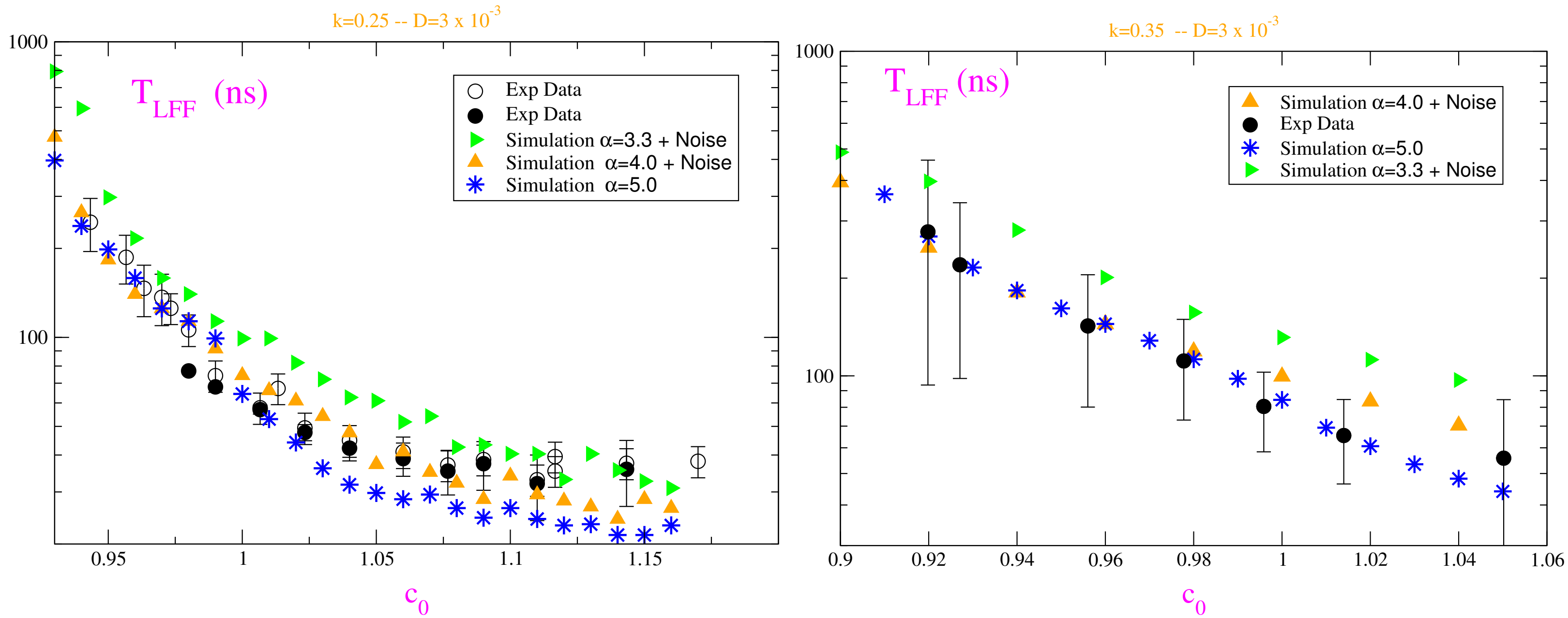
Lyapunov analysis



For noise variance $D > D_{exp}$ the asymptotic dynamics exhibits a positive maximal Lyapunov exponent for any examined α-values.

Experiments vs numerics

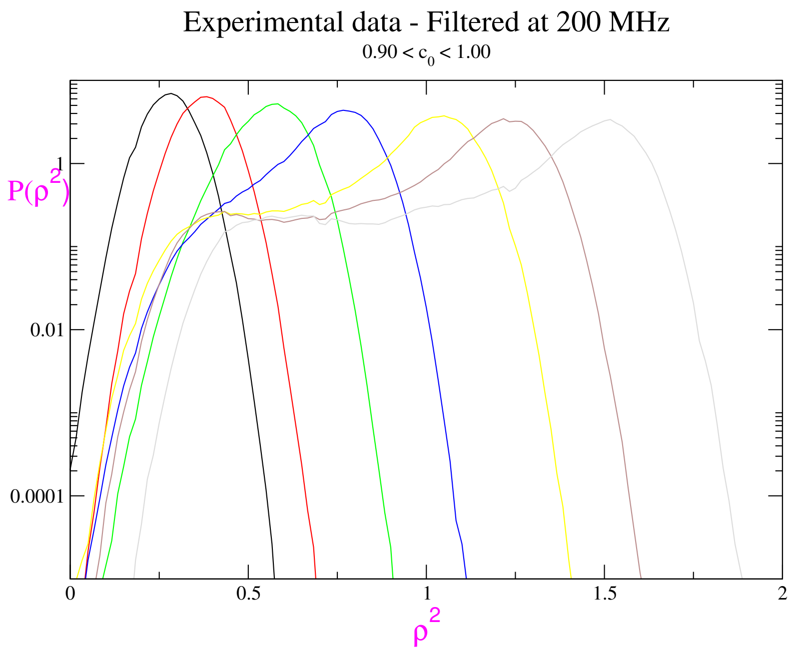
As a first comparison between experimental and numerical data the average duration of LFF $< T_{LFF} >$ is considered in the range $0.9 < c_0 < 1.1$ for two sets of experimental data .

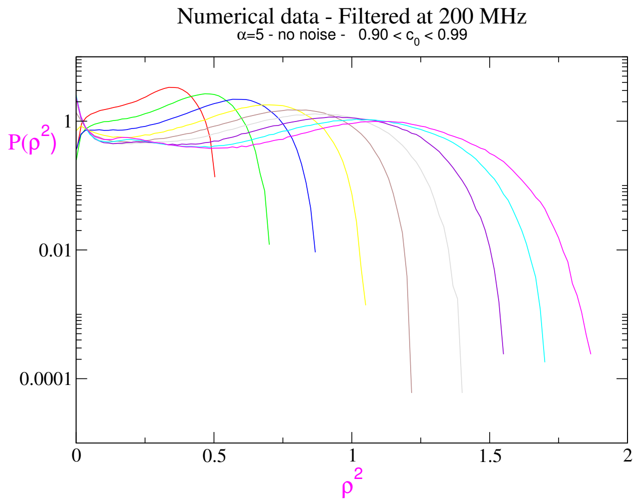


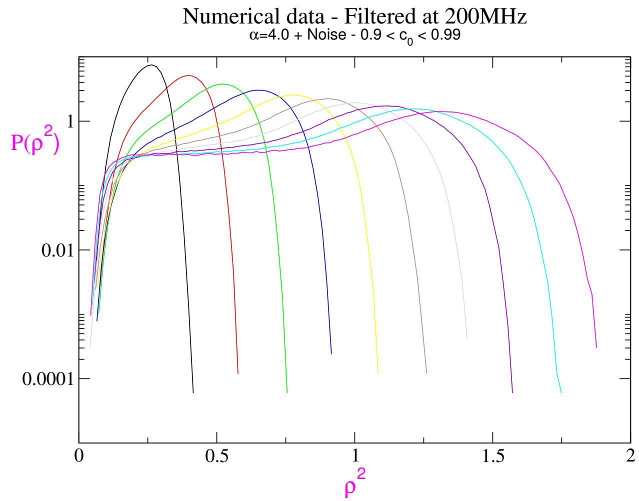
The agreemen is reasonably good for

- high values of $\alpha \sim 5$ without noise
- $\alpha \sim 3.3 - 4$ with noise $(D = D_{exp})$

PDFs of the Intensities



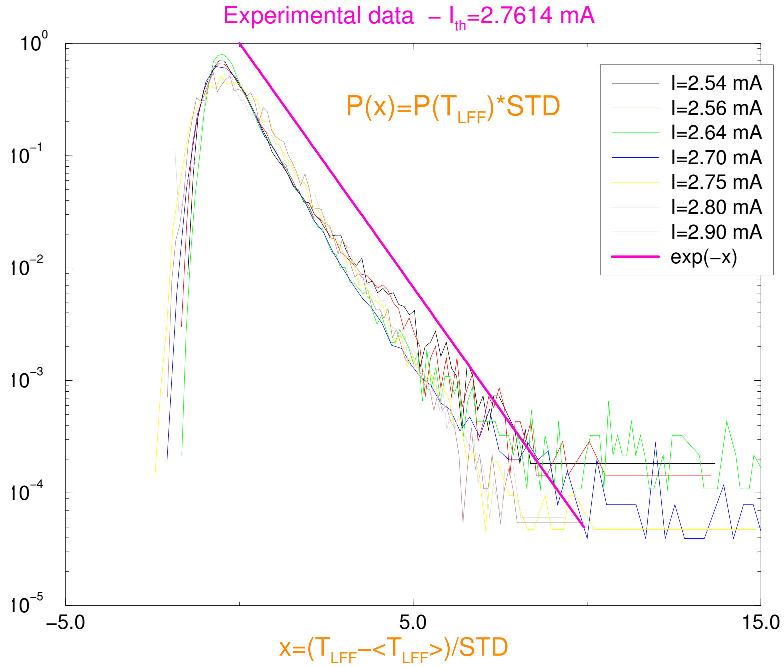


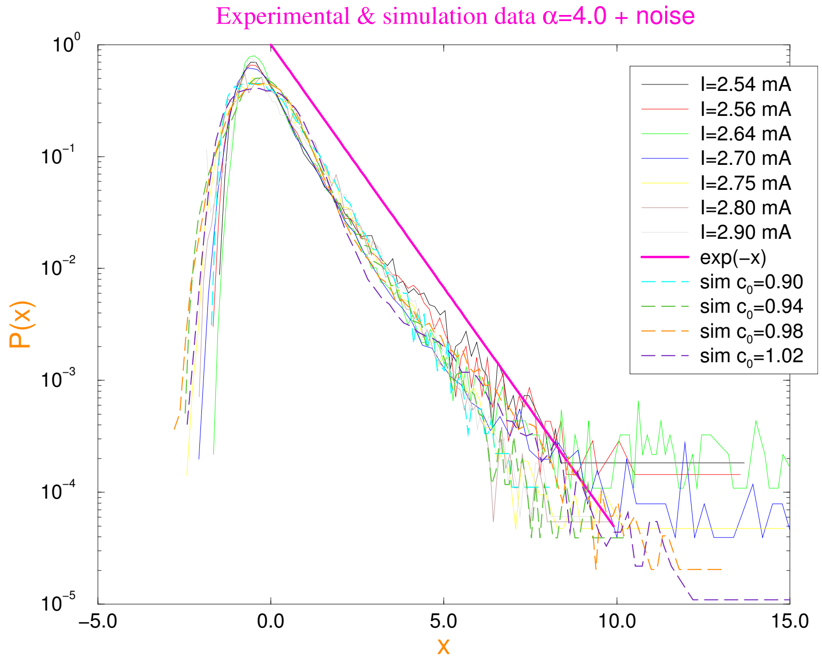


PDFs of the LFF times

$$P(x) = P(T_{LFF}) * \delta \quad x = \frac{T_{LFF} - \beta}{\delta}$$

where β is the average of T_{LFF} and $\delta = STD(T_{LFF})$.





First passage times

A Brownian motion with a drift can be written as

$$\dot{x}(t) = \mu + \sigma\xi(t)$$

with initial condition $x(0) = x_0$, $\xi(t)$ is a Gaussian noise with zero average.

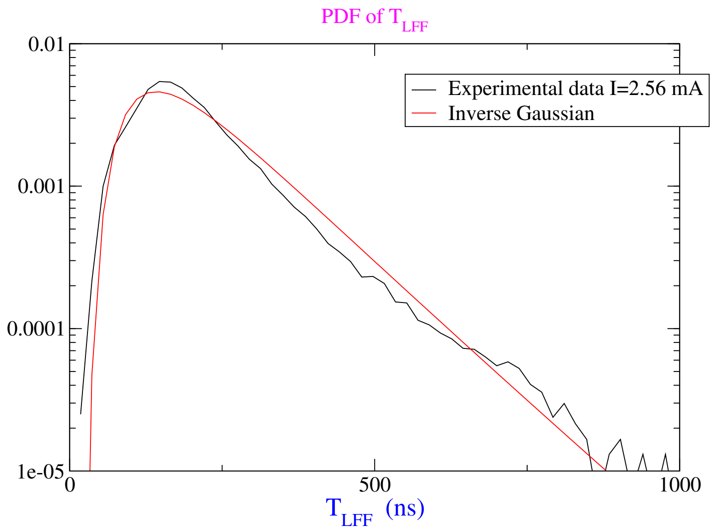
The average first passage time to reach a fixed threshold Θ is $\beta = (\Theta - x_0)/\mu$ and its variance is $\delta^2 = [(\Theta - x_0)\sigma^2]/\mu^3$.

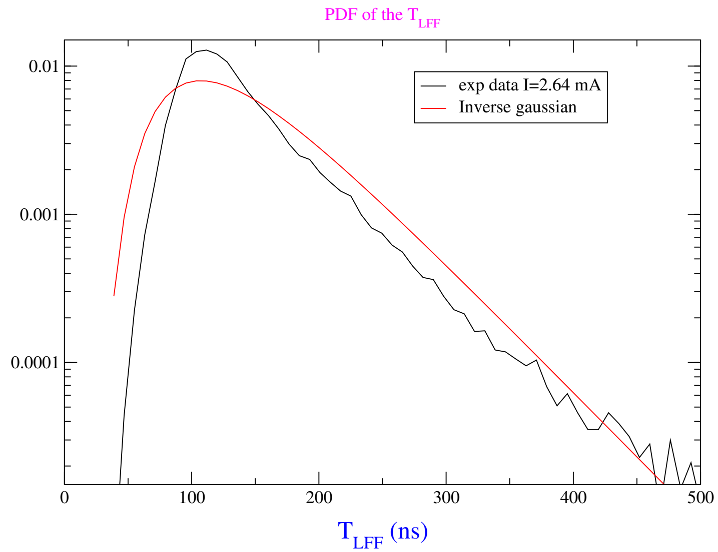
The PDF of the first passage times is the so-called inverse Gaussian :

$$P(T) = \frac{\beta}{\sqrt{2\pi\gamma T^3}} \mathrm{e}^{-(T-\beta)^2/(2\gamma T)}$$

where $\gamma = \delta^2/\beta$.

Comparison with the experiments





The inverse Gaussian describes reasonably well the experimental distributions

Concluding Remarks

- The deterministic LK eqs exhibit LFF as a transient chaotic dynamics;
- the introduction of additive noise in the LK eqs leads via intermittency to sustained LFF;
- the experimentally observed Sisypho Cycles can be interpreted as a Brownian motion with drift plus a reset mechanism ;
- the role of noise appears to be essential in the modelization of the phenomenon.