Frustrated two-dimensional quantum antiferromagnets in high magnetic fields & macroscopic quantum effects

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Plaquette spin models

Hamiltonian:

$$H = \frac{1}{2^n} \sum_\alpha J \vec{L}_\alpha^2 - h L_\alpha^z$$

$\vec{L}_\alpha = \sum_{i \in \alpha} \vec{S}_i$: total spin in plaquette α

2D	n	3D	n
Kagomé	1	garnet	1
checkerboard	1	pyrochlore	1
frustrated square lattice at $J' = J/2$	2		

classical ground state: $h = 0$: $\vec{L}_\alpha = 0 \quad \forall \alpha$

$h \neq 0$: $L_\alpha^z = \frac{h}{2J}$, $L_\alpha^x = L_\alpha^y = 0$

$\Rightarrow$ **large groundstate degeneracies**

$\Rightarrow$ perturbative spinwave theory diverges

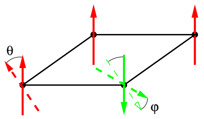
Collinear states

appear at special magnetic fields:

'uuud'	frustrated square lattice, checkerboard, pyrochlore	$\langle M \rangle = 1/2$
'uud'	Kagomé, garnet	$\langle M \rangle = 1/3$

Zero & soft modes

zero mode: tilt line of ↑↓↑↓↑↓ ...-spins by angle $\theta \perp h$



now turn ↓-spins around field axis by angle φ $\Rightarrow$ $E \sim \theta^2 \varphi^2$

$\Rightarrow$ **soft modes** (one for each zero mode !)

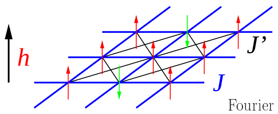
$\Rightarrow$ uuud-state: unusually many low-energy excitations

$\Rightarrow$ thermal fluctuations stabilize collinear states in classical model

'Order by disorder'

Frustrated square lattice

uuud-state at $J' = J/2$, $\langle M \rangle = 1/2$

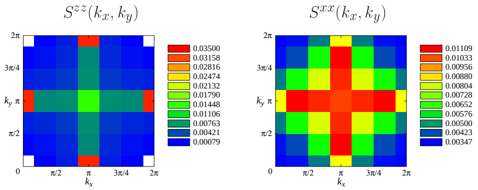


Fourier components: $(0,0)$, (π,π), $(0,\pi)$, $(\pi,0)$

$S = 1/2$ frustrated square lattice

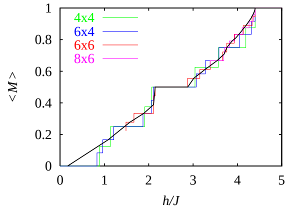
Static structure factors

$J'/J = 0.6$, $\langle M \rangle = 1/2$, 6×6 cluster

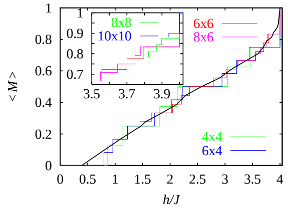


Magnetization curves

$J'/J = 0.6$:



$J'/J = 1/2$:



Value of the spin gap taken from Kotov *et al.*, Phys. Rev. **B60**, 14613 (1999)

One-magnon dispersion:

$$\mathcal{E}_{1s}(k_x, k_y) = -2(J + J') + J(\cos k_x + \cos k_y) + 2J' \cos k_x \cos k_y$$

$\Rightarrow$ one flat direction at $J' = J/2$: $k_x = \pi$ or $k_y = \pi$

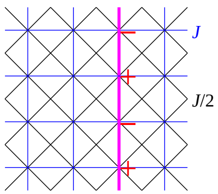
$\Rightarrow$ localization along x- or y-direction

$\Rightarrow$ finite-size jump ($\delta\langle M \rangle = 1/L$) on rectangle (short edge L)

$\Rightarrow$ transition to saturation exceptionally steep at $J' = J/2$

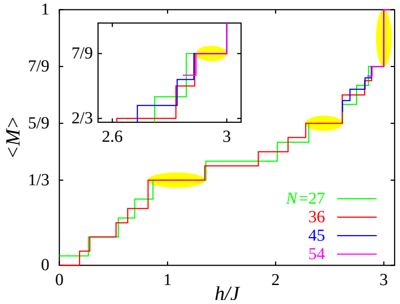
$\Rightarrow$ non-analytic asymptotic form, e.g.

$$1 - \langle M \rangle \sim \frac{1}{\log(h_{\text{sat}} - h)} \quad ?$$



$S = 1/2$ Kagomé lattice

Magnetization curve



- jump below saturation: $\delta\langle M \rangle = 2/9$
- plateaux with $\langle M \rangle = 7/9, 1/3, 5/9$ (?), ...

Localized magnons

close to saturation

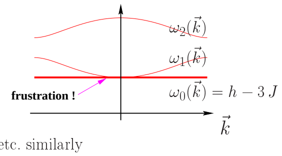
- $h \gg J$ $\Rightarrow$ $|\uparrow\uparrow\uparrow \ldots \uparrow\uparrow\uparrow\rangle$ ground state
- 1 magnon: $|\uparrow\downarrow\uparrow\uparrow\uparrow\uparrow \ldots\rangle$ $\rightarrow$ $|\uparrow\uparrow\uparrow\uparrow\downarrow\uparrow \ldots\rangle$

$S_{\vec{x}}^+ S_{\vec{y}}^-$

- **Kagomé**: basis △ & $\sum e^{i\vec{k}\cdot\vec{x}}$

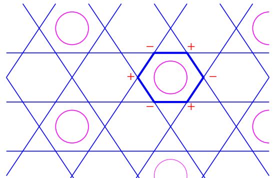
$\Rightarrow$ H: 3×3 matrix

$\Rightarrow$



pyrochlore etc. similarly

- inverse Fourier trafo $\Rightarrow$ localization in finite volume
- distance $\Rightarrow$ no interaction
- $\Rightarrow$ exact ground states
- finite density $\Rightarrow$ jump in $\langle M \rangle(h)$
- Max. # magnons: crystal (ordered) $\Rightarrow$ plateau in $\langle M \rangle(h)$



Kagomé: $\langle M \rangle = 1/3$ state

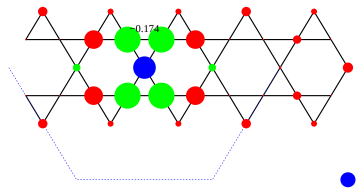
Classical spins ($S = \infty$):

no selection in collinear uud-manifold $\Rightarrow$ spin-liquid

Zhitomirsky, Phys. Rev. Lett. **88**, 057204 (2002)

Quantum spins: $S = 1/2$, $N = 36$ cluster

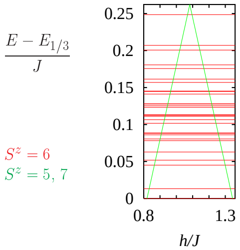
Correlations



• $\langle \vec{S}_0 \cdot \vec{S}_{\vec{y}} \rangle > 0$ • $\langle \vec{S}_0 \cdot \vec{S}_{\vec{y}} \rangle < 0$ • site $\vec{x} = 0$

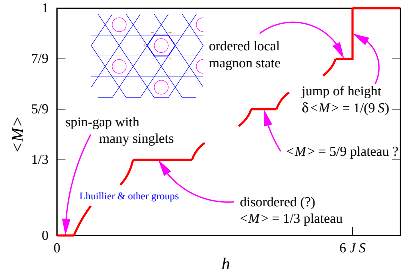
radius $\sim |\langle \vec{S}_0 \cdot \vec{S}_{\vec{y}} \rangle|$

Some excitations



$\Rightarrow$ spin-liquid with short-range uud-order (?)

Schematic magnetization curve



- many frustrated spin systems (3D pyrochlore, 1D models, ...): similar behaviour
- polarized state $|\uparrow\uparrow\uparrow \ldots \uparrow\uparrow\uparrow\rangle$: entropy = 0

demagnetization $h\downarrow$, at transition through h_{sat}: entropy ↑

$\Rightarrow$ T falls

$\Rightarrow$ **cooling by adiabatic demagnetization**

(compare Zhitomirsky, cond-mat/0210288)

Collaborators: D.C. Cabra, M.D. Grynberg, P.C.W. Holdsworth, O.A. Petrenko, P. Pujol, J. Richter, H.-J. Schmidt, J. Schnack, J. Schulenburg, M.E. Zhitomirsky

References: M.E. Zhitomirsky, A.H., O.A. Petrenko, Phys. Rev. Lett. **85**, 3269 (2000); A.H., Can. J. Phys. **79**, 1557 (2001); A.H., O.A. Petrenko, M.E. Zhitomirsky, Physica **B312-313**, 609 (2002); J. Schulenburg, A.H., J. Schnack, J. Richter, H.-J. Schmidt, Phys. Rev. Lett. **88**, 167207 (2002)