Exact ground states of highly frustrated quantum antiferromagnets in high magnetic fields

Festkörper-Theorie

cond-mat/0108498
to appear in PRL

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Construction

Hamiltonian

$$H = \sum_{\langle ij \rangle} J_{ij} \left\{ \Delta S_i^z S_j^z + \frac{1}{2} \left(S_i^+ S_j^- + S_i^- S_j^+ \right) \right\} - h \sum_i S_i^z$$

$\vec{S}_i$ spin-S operator at site i
$\langle ij \rangle$ nearest neighbours among the N sites

Localized one-magnon excitations

- Sufficiently high magnetic field
  $\Rightarrow$ ferromagnetically polarized state $|0\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$
- One-magnon state

$$|1\rangle \sim \sum_i a_i S_i^- |0\rangle$$

- Highly frustrated magnets: Lowest magnon dispersion $w_0(\vec{k})$ (out of n_s bands) independent (of some components) of $\vec{k}$ 'flat direction'
- Fourier transformation $\Rightarrow$ magnon(s) can be localized along flat directions in real space
- Dispersion completely flat $\Rightarrow$ localization in finite region

Non-interacting magnons

- Localized magnons with sufficiently large separation
  $\Rightarrow$ no interaction
  - Local magnon state: $a_l \neq 0$
  - Rest of the lattice: $a_r = 0$
  - Decoupling condition

$$\sum_l a_l J_{lr} = 0$$

  $\Rightarrow$ exact eigenstates
- **Theorem:** i) $S = 1/2$, $J_{ij} = J > 0$, $\Delta \geq 0$ or
  ii) $J_{ij} \geq 0$, $\Delta = 1$
  $\Rightarrow$ These states are lowest in energy, *i.e.* ground states
  i) Schnack, Schmidt, Richter, Schulenburg, EPJ **B24** (2001) 475
  ii) Schmidt, unpublished
- Linear $E(\langle M \rangle)$ $\Rightarrow$ jump in $\langle M \rangle (h)$
- Dispersion completely flat
  $\Rightarrow$ jump persists in thermodynamic limit

Independent magnons, finite density $\Rightarrow$ jump in magnetization curve

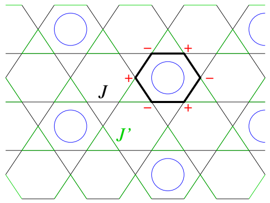
Maximal number of localized magnons: Solid state

$\Rightarrow$ magnetization plateaux

Two dimensions

Spin-S Kagomé lattice

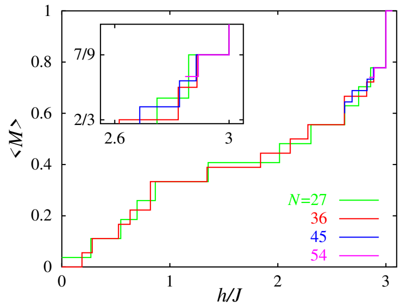
$w_0(\vec{k}) = h - S(J + J')(1 + 2\Delta)$ completely flat



Independent magnons coefficients of one-magnon state

Magnetization curve

$S = 1/2$, $\Delta = 1$, $J' = J$



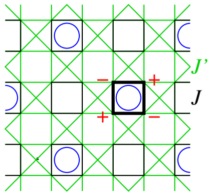
$\Rightarrow$ Jump below saturation: $\delta\langle M \rangle = 2/9$

$\Rightarrow$ Plateaux with $\langle M \rangle = 7/9, 1/3, 5/9$ (?), ...

Checkerboard

$w_0(\vec{k}) = h - S(J + J')(1 + 3\Delta)$ completely flat

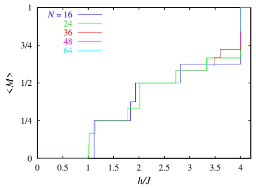
$J' \leq J$



Independent magnons
coefficients of one-magnon state

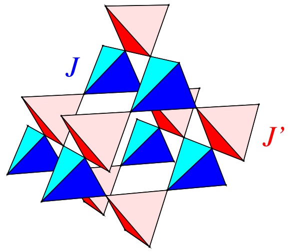
Magnetization curve

$S = 1/2$, $\Delta = 1$, $J' = J$



$\Rightarrow$ Jump below saturation: $\delta\langle M \rangle \geq 1/4$

Three dimensions: Pyrochlore



Lowest 2 (out of 4) magnon bands: degenerate and completely flat

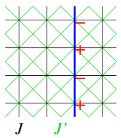
$$w_0(\vec{k}) = w_1(\vec{k}) = h - S(J + J')(1 + 3\Delta)$$

$\Rightarrow$ macroscopic jump $\delta\langle M \rangle \geq 1/6$ $\forall\, J, J' \geq 0$

Incomplete localization

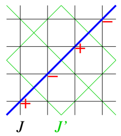
Two-dimensional lattices

Frustrated square $J' = J/2$ — localization along x- or y-direction



$k_x = \pi$ or $k_y = \pi$

Checkerboard $J' > J$ — localization along diagonals



$k_x \pm k_y = \pi$

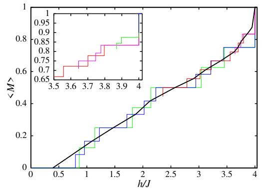
$\Rightarrow$ one flat direction

$\Rightarrow$ finite-size jump of vanishing height in thermodynamic limit

Frustrated square lattice

Magnetization curve

$S = 1/2$, $\Delta = 1$, $J' = J/2$



System sizes: 4×4, 6×4, 6×6, 8×6 (Inset: 8×8, 10×10)
Value of the spin gap taken from Kotov *et al.*, Phys. Rev. **B60** (1999) 14613

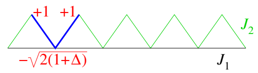
$\Rightarrow$ transition to saturation special in thermodynamic limit

$\Rightarrow$ non-analytic asymptotic form (like $1 - \langle M \rangle \sim \frac{1}{\log(h_{\text{sat}} - h)}$) ?

(see also A.H., Can. J. Phys. **79** (2001) 1557)

One-dimensional models

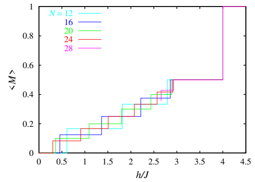
Sawtooth chain



$J_2 = \sqrt{2(1+\Delta)} J_1$ $\Rightarrow$ lowest magnon branch completely flat

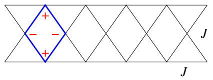
Magnetization curve

$S = 1/2$, $\Delta = 1$, $J_2 = 2J_1 = 2J$



$\delta\langle M \rangle = \frac{1}{2}$

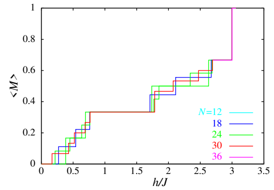
Kagomé-like chain I



Completely flat lowest magnon branch $w_0(k) = h - 2SJ(1 + 2\Delta)$

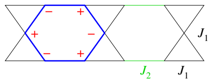
Magnetization curve

$S = 1/2$, $\Delta = 1$



$\delta\langle M \rangle = \frac{1}{3}$

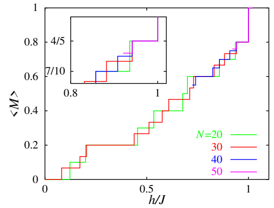
Kagomé-like chain II



$J_2 = \frac{(2\Delta+1)J_1}{\Delta+1}$ $\Rightarrow$ $w_0(k) = h - 2SJ_1(1 + 2\Delta)$ completely flat

Magnetization curve

$S = 1/2$, $\Delta = 0$, $J_1 = J_2 = J$



$\delta\langle M \rangle = \frac{1}{5}$