UNIVERSITÄT BONN
Physikalisches Institut

Representations of $N=1$ Extended Superconformal Algebras with Two Generators

W. Eholzer, A. Honecker, R. Hübel

published in: Mod. Phys. Lett. **A8** (1993) 725-738

Abstract

In this paper we consider the representation theory of $N = 1$ Super-$\mathcal{W}$-algebras with two generators for conformal dimension of the additional superprimary field between two and six. In the superminimal case our results coincide with the expectation from the ADE-classification. For the parabolic algebras we find a finite number of highest weight representations and an effective central charge $\tilde{c} = \frac{3}{2}$. Furthermore we show that most of the exceptional algebras lead to new rational models with $\tilde{c} > \frac{3}{2}$. The remaining exceptional cases show a new 'mixed' structure. Besides a continuous branch of representations discrete values of the highest weight exist, too.

Post address:
Nußallee 12
W-5300 Bonn 1
Germany
e-mail: ralf@mpim-bonn.mpg.de

BONN-HE-92-28
hep-th/9209030
Bonn University
September 1992
ISSN-0172-8733

1. Introduction

One of the most interesting questions in two-dimensional conformal quantum field theory is the classification of all rational models. So far no general classification schemes leading to a complete list of all RCFTs have been developped. One hopes to describe all rational models as minimal models of the symmetry algebra of a conformally invariant theory, which therefore contains the Virasoro algebra as a subalgebra. Consequently a reasonable ansatz is the construction of extensions of the conformal algebra, $\mathcal{W}$-algebras, and the study of their highest weight representations.

After the initial work of Zamolodchikov in 1985 [32] several methods have been found to construct $\mathcal{W}$-algebras. Besides the GKO-construction [20][3][6][1][2] and the free field approach [15][14] a method constructing the algebra explicitly exists [9][25], which we call the Lie bracket approach. Another equivalent method is the conformal bootstrap [4][16][10]. Starting with the results of [9][31] we investigated the representation theory of $\mathcal{W}(2,d)$-algebras [12].

With the methods cited above several $N=1$ supersymmetric $\mathcal{W}$-algebras have been constructed [17][23][7]. Recently we constructed a lot of new of $N=1$ $\mathcal{SW}$-algebras with two and three generators using the non-covariant Lie bracket approach [8]. These results led to a better understanding of the classification of $N=1$ $\mathcal{SW}$-algebras. In this paper we use the same methods as in [12] to investigate the representation theory of the $\mathcal{SW}(\frac{3}{2},\delta)$-algebras with conformal dimension of the additional generator between two and six. We stress the fact that our methods yield necessary conditions for the existence of consistent highest weight representations.

In the next chapter we recall the notions and the results of the construction of $N=1$ $\mathcal{SW}$-algebras. We proceed with a chapter concerning the representation theory of $\mathcal{SW}$-algebras and discuss some new problems arising in the Ramond sector. In chapter four we present the explicit results of our calculations. We end with a summary in the last chapter.

2. Notations and previous results

Let $\mathcal{F}$ be the algebra of local chiral fields of conformal field theory defined on a two-dimensional compactified spacetime. The requirement of invariance under rational conformal transformations induces a grading of $\mathcal{F}$ by the conformal dimension. We assume that $\mathcal{F}$ is spanned by the quasiprimary fields and derivatives thereof. Furthermore $\mathcal{F}$ carries the (non-associative) operation of forming normal ordered products. Let $\phi(z) := \sum_{n-d(\phi)\in\mathbb{Z}} z^{n-d(\phi)}\phi_n$ be the Fourier decomposition of the chiral field $\phi(z)$ with conformal dimension $d(\phi)$ and call ϕ_n the modes of ϕ. It is well known that the modes $\{L_n | n\in\mathbb{Z}\}$ of the energy momentum tensor satisfy the Virasoro algebra. A primary field ϕ obeys the following commutation relations:

$$[L_n,\phi_m] = \big(m-(d(\phi)-1)n\big)\phi_{n+m} \tag{2.1}$$

If (2.1) is valid only for $n\in\{-1,0,1\}$, ϕ is a quasiprimary field.

W. Nahm showed that the requirements of locality and invariance under rational conformal transformations determine the commutator of two chiral quasiprimary fields up to some structure constants which are determined by additional dynamical principles (i.e. Jacobi identities) [29]. Let $\{\phi_i \mid i \in I\}$ be a set of quasiprimary chiral fields with conformal dimensions $d(\phi_i) \in \mathbb{N}/2$ which together with their derivatives span $\mathcal{F}$. The commutator of the modes of the quasiprimary fields is given by

$$[\phi_{i,m}, \phi_{j,n}]_\pm = \sum_{k\in I} C_{ij}^k \, p_{ijk}(m,n)\phi_{k,m+n} + d_{ij}\, \delta_{n,-m} \binom{n + d(\phi_i) - 1}{2d(\phi_i) - 1} \tag{2.2}$$

d_{ij} describes the normalization of the two point functions and C_{ij}^k are coupling constants. p_{ijk} are universal polynomials depending only on the conformal dimensions of the fields involved (for details see [9]).

The Fourier modes of the normal ordered product of two chiral fields are defined by

$$N(\phi,\psi)_n := (-1)^{4d(\phi)d(\psi)} \sum_{k<d(\psi)} \phi_{n-k}\psi_k \; + \sum_{k\geq d(\psi)} \psi_k\phi_{n-k} \tag{2.3}$$

To make use of the commutator formula (2.2) one has to consider the quasiprimary projection of (2.3):

$$\mathcal{N}(\phi_j, \partial^n\phi_i) := N(\phi_j, \partial^n\phi_i) - \big(\sum_{r=1}^{n} \alpha_{ij}^r \partial^r N(\phi_j, \partial^{n-r}\phi_i) + \sum_k \beta_{ij}^k(n) C_{ij}^k \partial^{\gamma_{ij}^k(n)}\phi_k\big) \tag{2.4}$$

$\alpha_{ij}^r, \beta_{ij}^k(n), \gamma_{ij}^k(n)$ are polynomials depending only on the conformal dimensions of the fields involved [9]. This definition yields a quasiprimary field of conformal dimension $d(\phi_i) + d(\phi_j) + n$.

As the number of quasiprimary fields in $\mathcal{F}$ is infinite we introduce the notion of 'simple' fields. Let $\mathcal{B} = \{\phi_i \mid i \in I\}$ be a subset of the quasiprimary fields of $\mathcal{F}$. $\mathcal{B}$ generates $\mathcal{F}$ if it is possible to obtain a (vectorspace) basis of $\mathcal{F}$ out of $\mathcal{B}$ using the operations of forming derivatives and normal ordered products. If the fields in $\mathcal{B}$ are orthogonal to all normal ordered products in $\mathcal{F}$ we call them 'simple' fields.

The super Virasoro algebra is the extension of the Virasoro algebra by a primary simple field $G(z)$ of conformal dimension $\frac{3}{2}$. It is a Lie superalgebra and exists for generic values of the central charge c. With the standard normalization $d_{\phi\phi} = \frac{c}{d(\phi)}$ of a quasiprimary field ϕ with conformal dimension $d(\phi)$ the commutation relations take the following form:

$$\begin{aligned}
[L_m, L_n] &= (n-m)L_{m+n} + \tfrac{c}{12}(n^3 - n)\delta_{n+m,0} \\
[L_m, G_n] &= (n - \tfrac{1}{2}m)G_{m+n} \\
[G_m, G_n]_+ &= 2L_{m+n} + \tfrac{c}{3}(m^2 - \tfrac{1}{4})\delta_{m+n,0}
\end{aligned} \tag{2.5}$$

This algebra is the central extension of the algebra formed by the generators of superconformal transformations in superspace Z, consisting of the points (z, θ) where θ is a

Grassmannian variable. A field $\Phi(Z) = \phi(z) + \theta\psi(z)$ is called super primary with superconformal dimension $\delta = d(\Phi)$, iff ϕ and ψ are Virasoro primaries with conformal dimension δ resp. $\delta + \frac{1}{2}$ and satisfy the following commutation relations

$$\begin{aligned}
[G_n, \phi_m]_\pm &= C^{\psi}_{G\phi} p_{\frac{3}{2},\delta,\delta+\frac{1}{2}}(n,m)\ \psi_{n+m} = C^{\psi}_{G\phi}\psi_{n+m} \\
[G_n, \psi_m]_\pm &= C^{\phi}_{G\psi} p_{\frac{3}{2},\delta+\frac{1}{2},\delta}(n,m)\ \phi_{n+m} = \frac{C^{\phi}_{G\psi}}{2\delta}(m - (2\delta - 1)n)\phi_{n+m}
\end{aligned} \tag{2.6}$$

$C^{\psi}_{G\phi}$ and $C^{\phi}_{G\psi}$ are special known structure constants [8]. If ϕ and ψ are quasiprimaries and (2.6) is fulfilled only for $n \in \{-\frac{1}{2}, \frac{1}{2}\}$, Φ is called a super quasiprimary.

Definition: Let $\mathcal{B} = \{G, L, \phi_1, \psi_1, \ldots, \phi_n, \psi_n\}$ be a set of simple fields with the additional property that $\Phi_i = \phi_i + \theta\psi_i, i = 1, \ldots, n$ are super primaries. The associative field algebra generated by $\mathcal{B}$ is called a $\mathcal{SW}\big(\frac{3}{2}, d(\Phi_1), \ldots, d(\Phi_n)\big)$.

For a $\mathcal{SW}(\frac{3}{2}, \delta_1, \ldots, \delta_n)$-algebra to be consistent it is necessary that all Jacobi identities involving the additional primary fields are fulfilled. In [8] we described in detail how to construct the algebras using the Lie bracket approach. Let us recall the results of the construction of $\mathcal{SW}(\frac{3}{2}, \delta)$-algebras. The central charge and the conformal dimensions of the super primaries of the minimal models of the super Virasoro algebra are given by the following relations [5][13][19][27][28]:

$$\begin{aligned}
c(p,q) &= \tfrac{3}{2}\big(1 - \tfrac{2(p-q)^2}{pq}\big)\ p,q \in \mathbb{N}, \gcd(p,q) = 1, p+q \in 2\mathbb{N} \vee p,q \in 2\mathbb{N}, \gcd(\tfrac{p}{2},\tfrac{q}{2}) = 1, \tfrac{p}{2}+\tfrac{q}{2} \not\in 2\mathbb{N} \\
h(r,s) &= \tfrac{(rp-qs)^2-(p-q)^2}{8pq} + \tfrac{1-(-1)^{r+s}}{32} \quad 1 \le r \le q-1\ ,\ 1 \le s \le p-1
\end{aligned} \tag{2.7}$$

$r+s$ even yields representations in the Neveu-Schwarz sector and $r+s$ odd representations in the Ramond sector. In perfect analogy to $\mathcal{W}(2,d)$-algebras the c-values for which the $\mathcal{SW}(\frac{3}{2}, \delta)$-algebras are consistent can be divided in five classes.

- generically existing $\mathcal{SW}(\frac{3}{2}, \delta)$-algebras
- algebras existing for superminimal values of the central charge and therefore being related to the models of the ADE-classification of A. Cappelli et al. [11]
- algebras existing for central charges of the form $c = c(1,s)$, $2 \le s \in \mathbb{N}$
- parabolic algebras related to degenerate models of the super Virasoro algebra
- exceptional algebras which cannot be put in any other class

In the following table we recall the concrete values of the central charge for the $\mathcal{SW}$-algebras $\mathcal{SW}(\frac{3}{2}, \delta)$ with $\frac{3}{2} \le \delta \le 6$ and their classification into the different classes.

δ	c	series	c	series	c	series
2	$-\frac{6}{5}$ $(C_{\Phi\Phi}^{\Phi}=0)$	(A_3, D_6)	generic $(C_{\Phi\Phi}^{\Phi} \neq 0)$			
$\frac{5}{2}$	$-\frac{5}{2}$	$(1, s)$	$\frac{10}{7}$	(D_8, E_6)		
3	$-\frac{27}{7}$	(A_3, D_8)	$\frac{5}{4}$	(A_7, D_4)	$-\frac{45}{2}$	parabolic
$\frac{7}{2}$	$\frac{7}{5}$	(A_9, E_6)	$-\frac{17}{11}$	exceptional		
4	$-\frac{20}{3}$	(A_3, D_{10})	$-\frac{21}{2}$	parabolic		
	$-\frac{185}{4}$	exceptional	-13	exceptional	$-\frac{120}{13}$	exceptional
$\frac{9}{2}$	$-\frac{81}{10}$	$(1, s)$	$\frac{4}{11}$	(D_{12}, E_6)	$-\frac{69}{2}$	parabolic
5	$-\frac{105}{11}$	(A_3, D_{12})				
$\frac{11}{2}$	$-\frac{5}{13}$	(D_{14}, E_6)	$\frac{10}{7}$	(A_{13}, E_6)	$\frac{11}{40}$	(A_{15}, E_8)
	$-\frac{705}{8}$	exceptional	$-\frac{155}{19}$	exceptional		
6	$-\frac{162}{13}$	(A_3, D_{14})	$\frac{27}{20}$	(A_7, D_6)	$-\frac{93}{2}$	parabolic
	$-\frac{33}{2}$	parabolic	-18	exceptional	$-\frac{2241}{20}$	exceptional

3. Representations of $\mathcal{SW}(\frac{3}{2}, \delta)$ algebras

Per definition $\mathcal{SW}$-algebras contain fermionic fields, i.e. fields with half-integer conformal dimension. In the representation theory of these algebras we have to distinguish between the Neveu-Schwarz and the Ramond sector. From the mathematical point of view fermionic fields are defined on a double sheet covering space of the complex plane and therefore have nontrivial monodromy properties. In the Neveu-Schwarz (NS) and Ramond (R) sector a fermionic field $\Omega(z)$ obeys the following transformation laws under the transformation $z \to e^{2\pi i} z$:

$$\Omega(e^{2\pi i} z) = \Omega(z) \qquad \text{NS sector}$$
$$\Omega(e^{2\pi i} z) = -\Omega(z) \quad \text{R sector}$$

Consequently fermionic fields have integer modes in the Ramond sector and half-integer modes in the Neveu-Schwarz sector.

In the following we will consider $\mathcal{SW}$-algebras with one additional generator $\Phi = \phi + \theta\psi$. Let w.l.o.g. δ be an integer and denote the algebra $\mathcal{SW}(\frac{3}{2}, \delta)$ by $\mathcal{A}$.

Definition: The abelian two dimensional subalgebra $\mathcal{C}$ of $\mathcal{A}$ generated by the zero modes of the bosonic simple fields (L_0, ϕ_0) is called the Cartan subalgebra of $\mathcal{A}$. The subalgebra of $\mathcal{A}$ which is generated by the zero modes of all fields is called the 'horizontal' subalgebra.

Definition: A linear representation (V, λ), $\lambda \in \mathcal{C}^*$ of $\mathcal{A}$ is a highest weight representation (HWR) iff the complex vectorspace V contains a nonzero unique cyclic vector $\mid h, w \rangle$ with the following properties:

$$\begin{aligned}
L_0 \ \mid h,w\,\rangle = \lambda(L_0) \mid h,w\,\rangle \ &= \ h \mid h,w\,\rangle \\
\phi_0 \ \mid h,w\,\rangle = \lambda(\phi_0) \mid h,w\,\rangle \ &= \ w \mid h,w\,\rangle \\
L_n \ \mid h,w\,\rangle = G_n \ \mid h,w\,\rangle = \phi_n \ \mid h,w\,\rangle &= \psi_n \ \mid h,w\,\rangle = 0 \quad \forall n<0
\end{aligned} \tag{3.1}$$

The representation module V is spanned by the vectors which are obtained by applying the positive modes of the simple fields to the highest weight vector (HWV) $\mid h,w\,\rangle$.

In the Neveu-Schwarz sector the representation theory of $\mathcal{SW}$-algebras is very similar to that of ordinary fermionic $\mathcal{W}$-algebras. But in the Ramond sector it is more complicated since here the zero modes of the fermionic simple fields are involved. First of all we establish that the ground state in the representation module generally possesses a fourfold degeneracy because the vectors $\mid h,w\,\rangle$, $G_0 \mid h,w\,\rangle$, $\psi_0 \mid h,w\,\rangle$, $\psi_0 G_0 \mid h,w\,\rangle$ have all the same L_0-eigenvalue h. It is easy to see that there are no more linearly independent vectors with L_0-eigenvalue h. In several cases we checked explicitly that e.g. the vector $\psi_0 G_0 \mid h,w\,\rangle$ is not proportional to the HWV $\mid h,w\,\rangle$. The investigation of the representation theory of the horizontal subalgebra should lead to a better understanding of the phenomenon of the ground state degeneracy. For practical calculations this degeneracy has the consequence that in the evaluation of Jacobi identities (see below) the correlators $\langle\, h,w \mid G_0 \mid h,w\,\rangle$, $\langle\, h,w \mid \psi_0 \mid h,w\,\rangle$, $\langle\, h,w \mid \psi_0 G_0 \mid h,w\,\rangle$ appear as additional unknowns. Therefore we need in general one more linearly independent condition than in the Neveu-Schwarz sector to restrict the highest weight to a finite number of values (we evaluated only Jacobi identities which contained an even number of fermionic fields so that only the last one of the above correlators appeared).

The second difficulty arising in the Ramond sector is the definition of the quasiprimary normal ordered product $\mathcal{N}(\phi,\psi)$. The standard definition (2.4) fails in the Ramond sector if ψ is a fermionic field [22]. In the practical calculations we were able to surround this problem by using the commutator formula of W. Nahm (2.2). At first one avoids the occurence of NOPs of the form $\mathcal{N}(\phi,\psi)$ with $d(\phi) \in \mathbb{N}$ and $d(\psi) \in \mathbb{N}+\frac{1}{2}$ by choosing a special basis in the space of quasiprimary fields. It is possible to choose a basis where only NOPs of fermionic with bosonic fields occur with the bosonic field being the second entry of $\mathcal{N}(.,.)$. However, in this basis NOPs of two fermionic fields still appear. As an example consider the basis of quasiprimary fields of conformal dimension 4 constructed out of L and G which is given by $\mathcal{N}(L,L), \mathcal{N}(G,\partial G)$. To handle the second NOP we use the commutator formula (2.2) to express it by NOPs which can be evaluated with formula (2.4). With (2.2) we obtain

$$\begin{aligned}
[G_m\,,\mathcal{N}(G,L)_n]_+ = p_{\frac{3}{2},\frac{7}{2},2}(m,n)\; C^{L}_{G\mathcal{N}(G,L)}\; L_{m+n} \; + \\
p_{\frac{3}{2},\frac{7}{2},4}(m,n)\big(C^{\mathcal{N}(L,L)}_{G\mathcal{N}(G,L)}\;\mathcal{N}(L,L)_{m+n} + C^{\mathcal{N}(G,\partial G)}_{G\mathcal{N}(G,L)}\;\mathcal{N}(G,\partial G)_{m+n}\big)
\end{aligned} \tag{3.3}$$

Inserting the polynomials and structure constants [22] yields ($m=0$):

$$\mathcal{N}(G,\partial G)_n = 2\big(G_0\mathcal{N}(G,L)_n + \mathcal{N}(G,L)_n G_0 - 2\,\mathcal{N}(L,L)_n + \tfrac{1}{48}(4c+21)\,L_n\big) \tag{3.4}$$

Analogously one obtains formulae for the other NOPs with two fermionic fields. With this procedure we have been able to calculate the correct h-values for the HWRs in the Ramond sector at the expense of computer time.

Let us finally describe in short the methods we used to obtain the relevant HWRs.From the generically existing $\mathcal{SW}(\frac{3}{2},\delta)$-algebras we considered only the case $\delta = 2$ because in all other cases the algebras are Lie superalgebras for which the representation theory is well understood [30]. In complete analogy to the generically existing $\mathcal{W}(2,d)$-algebras we determined in the continuum of representations for special c-values rational models by constructing null fields. We stress that the evaluation of Jacobi identities on the HWV did not yield any restrictions in this case.

The $\mathcal{SW}(\frac{3}{2},\delta)$-algebras for $\delta \geq \frac{5}{2}$ exist only for discrete values of the central charge. Here the evaluation of Jacobi identities leads to restrictions of the highest weight. We considered the following three- and four-point functions (w.l.o.g. $\delta \in \mathbb{N}$):

$$
\begin{aligned}
0 = \; & \langle\, h\,,\, w \mid [[\phi_{-n},\phi_{-m}],\phi_{n+m}]_{cycl.} \mid h\,,\, w\,\rangle \\
0 = \; & \langle\, h\,,\, w \mid [[\phi_{-n},\psi_{-m}]_{\pm},\psi_{n+m}]_{\pm,cycl.} \mid h\,,\, w\,\rangle \\
0 = \; & \langle\, h\,,\, w \mid \phi_{-n}[[\phi_{-m},\phi_{n}],\phi_{m}]_{cycl.} \mid h\,,\, w\,\rangle \\
0 = \; & \langle\, h\,,\, w \mid \phi_{-n}[[\phi_{n},\psi_{-m}]_{\pm},\psi_{m}]_{\pm,cycl.} \mid h\,,\, w\,\rangle \\
0 = \; & \langle\, h\,,\, w \mid \psi_{-n}[[\psi_{-m},\psi_{n}]_{\pm},\psi_{m}]_{\pm,cycl.} \mid h\,,\, w\,\rangle
\end{aligned}
\tag{3.5}
$$

For a more detailed description of these methods see for example [12].

4. Results

We start with the results for the algebra $\mathcal{SW}(\frac{3}{2},2)$. This algebra is most probably the only generically existing $\mathcal{SW}$-algebra with two generators which is no Lie superalgebra. The Jacobi identities (3.5) lead to expressions which become trivial if one inserts the relation between the self coupling constant and c. Therefore we assert that this algebra possesses arbitrary HWRs, an assumption which is supported by the existence of a free field construction [26]. For the set of c-values $\{-\frac{6}{5},-\frac{39}{4},-\frac{9}{2},\frac{3}{2},33\}$ we constructed two null fields with conformal dimensions 4 and $\frac{9}{2}$ in each case (in the case $c = \frac{3}{2}$ with dimensions 4 and 5) [22]. Up to now it is an open question if a single null field yields enough linearly independent conditions to restrict the highest weight to a finite number of values. The value $c = -\frac{6}{5}$ lies in the minimal series of the super Virasoro algebra and the value $c = -\frac{9}{2}$ belongs to the parabolic cases. Below we give a general parametrization of the possible h-values for these classes. The remaining c-values belong to the class ‘exceptional’, so that we present the data explicitly ($q = w/C_{\phi\phi}^{\phi}$):

$\mathcal{SW}(\frac{3}{2},2)$			
c	$-\frac{39}{4}$	$\frac{3}{2}$	33
	(h,q)	(h,q)	(h,q)
NS	$\big((0,0),(-\frac{1}{4},\frac{1}{38})\big)\ \vee$ $q=\frac{1}{38}(20h+9)$	$\big((\frac{1}{2},\frac{1}{2}),(\frac{1}{8},\frac{1}{32})\big)\ \vee$ $q=-\frac{1}{2}h$	$q=\frac{1}{19}h\ \vee$ $q=\frac{2}{19}(2h-3)$
R	$(-\frac{1}{32},\frac{11}{152})\ \vee$ $p_0q^2-p_1q+p_2=0$	$\big((\frac{9}{16},-\frac{9}{32}),(\frac{9}{16},\frac{15}{32})\big)\ \vee$ $q=\frac{1}{32}(3-16h)$	$q=\frac{1}{19}h\ \vee$ $q=\frac{2}{19}(2h-3)$

with $p_0(h)=92416,\ p_1(h)=97280h+31008,\ p_2(h)=25600h^2+19776h+3861.$

The first two cases share the same structure of the HWRs. Besides a continuous one parameter branch of HWRs there further discrete values of h,q exist. We point out that this pattern appears also for c-values of the other two algebras under investigation with even integer superconformal dimension of the additional generator. Note that in the exceptional case $c=33$ the restrictions for both sectors coincide.

Let us now discuss the results of our calculations for $\mathcal{SW}(\frac{3}{2},\delta)$-algebras which exist for c-values in the minimal series of the super Virasoro algebra. We separate these values into four different series due to ADE-classification of the modular invariant partition functions built up by the characters of the correspondent minimal model [11]. For all these rational models one obtains $\tilde{c}=c-24h_{min}<\frac{3}{2}$, h_{min} being the smallest h-value of the possible representations.

The first series consists of the minimal c-values with the modular invariant partition function of the type $(A_{q-1},D_{\frac{p+2}{2}})$. The $\mathcal{SW}$-algebras belonging to this series have a vanishing self coupling constant. In complete analogy to the conformal case we can express the characters of the HWRs of the $\mathcal{SW}(\frac{3}{2},\delta)$-algebra as a finite sum of superconformal characters appearing in the modular invariant partition function. In other words the characters of the $\mathcal{SW}$-algebra diagonalize the partition function. In the NS sector the characters are given by ($\mathcal{I}_1=2\,\mathbb{N}+1,\ \mathcal{I}_2=2\,\mathbb{N}$):

$$\begin{aligned}\chi^{SW}_{i,j}&=\chi_{i,j}+\chi_{i,p-j},\quad 1\le i\le\tfrac{q}{2},\ 1\le j\le\tfrac{p}{2}-2,\ i,j\in\mathcal{I}_1\\ \chi^{SW}_{i,\frac{p}{2}}&=\chi_{i,\frac{p}{2}},\quad 1\le i\le\tfrac{q}{2},\ i\in\mathcal{I}_1\end{aligned}\tag{4.1a}$$

For $i\in\mathcal{I}_2$ one obtains the characters in the R sector. Of course one has to replace $\chi_{i,j}$ by $\hat{\chi}_{i,j}$. We stress the fact that the characters span a representation space of the subgroups of the modular group $SL(2,\mathbb{Z})$ generated by the elements S and T^2 (NS sector) respectively ST^2S and T (R sector). From (4.1a) we can read off immediately the h-values of the HWRs in the NS sector (parametrization due to (2.7)):

$$\begin{aligned}h^{SW}_{i,j}&=\min(h_{i,j},h_{i,p-j}),\quad 1\le i\le\tfrac{q}{2},\ 1\le j\le\tfrac{p}{2}-2,\ i,j\in\mathcal{I}_1\\ h^{SW}_{i,\frac{p}{2}}&=h_{i,\frac{p}{2}},\quad 1\le i\le\tfrac{q}{2},\ i\in\mathcal{I}_1\end{aligned}\tag{4.1b}$$

The second series is made up of the minimal c-values with a partition function of the type (A_{q-1},E_6). Realizations are provided by $\mathcal{SW}(\frac{3}{2},\frac{7}{2})$, $c=\frac{7}{5}$ and $\mathcal{SW}(\frac{3}{2},\frac{11}{2})$, $c=\frac{10}{7}$ with non-vanishing self coupling constant. In the NS sector the characters of the HWRs of the $\mathcal{SW}$-algebra can be written as follows:

$$\begin{aligned} &\chi^{SW}_{i,1} = \chi_{i,1} + \chi_{i,7},\ \chi^{SW}_{i,2} = \chi_{i,5} + \chi_{i,11}, \quad 1 \le i \le \tfrac{q}{2},\ i \in \mathcal{I}_1 \\ &\chi^{SW}_{i,3} = \chi_{i,4} + \chi_{i,8}, \quad 1 \le i \le \tfrac{q}{2},\ i \in \mathcal{I}_2 \end{aligned} \tag{4.2}$$

The characters in the R sector are obtained by interchanging $\mathcal{I}_1$ with $\mathcal{I}_2$.

The third family contains c-values related to the partition functions of the type $(D_{\frac{q}{2}+1}, E_6)$. We found three algebras with vanishing self coupling constant which fit into this pattern: $\mathcal{SW}(\frac{3}{2},\frac{5}{2})$, $c = \frac{10}{7}$, $\mathcal{SW}(\frac{3}{2},\frac{9}{2})$, $c = \frac{4}{11}$ and $\mathcal{SW}(\frac{3}{2},\frac{11}{2})$, $c = -\frac{5}{13}$. The characters are given by:

$$\begin{aligned} \text{NS sector: } \chi^{SW}_{i} &= \chi_{i,1} + \chi_{i,5} + \chi_{i,7} + \chi_{i,11}, \quad 1 \le i < \tfrac{q}{2},\ i \in \mathcal{I}_1 \\ \chi^{SW}_{\frac{q}{2}} &= \chi_{\frac{q}{2},1} + \chi_{\frac{q}{2},5} \\ \text{R sector: } \hat{\chi}^{SW}_{i} &= \hat{\chi}_{i,4} + \hat{\chi}_{i,8}, \quad 1 \le i < \tfrac{q}{2},\ i \in \mathcal{I}_1 \\ \hat{\chi}^{SW}_{\frac{q}{2}} &= \hat{\chi}_{\frac{q}{2},4} \end{aligned} \tag{4.3}$$

The last family consist of super minimal c-values connected with the (A_{q-1}, E_8) partition functions. Only one $\mathcal{SW}$-algebra with non-vanishing self coupling constant is constructed so far realizing the series: $\mathcal{SW}(\frac{3}{2},\frac{11}{2})$, $c = \frac{11}{40}$. The characters read:

$$\begin{aligned} \text{NS sector: } \chi^{SW}_{i,1} &= \chi_{i,1} + \chi_{i,11} + \chi_{i,19} + \chi_{i,29}, \quad 1 \le i \le \tfrac{q}{2},\ i \in \mathcal{I}_1 \\ \chi^{SW}_{i,2} &= \chi_{i,7} + \chi_{i,13} + \chi_{i,17} + \chi_{i,23}, \quad 1 \le i \le \tfrac{q}{2},\ i \in \mathcal{I}_1 \\ \text{R sector: } \hat{\chi}^{SW}_{i,1} &= \hat{\chi}_{i,1} + \hat{\chi}_{i,11} + \hat{\chi}_{i,19} + \hat{\chi}_{i,29}, \quad 2 \le i < \tfrac{q}{2},\ i \in \mathcal{I}_2 \\ \hat{\chi}^{SW}_{\frac{q}{2},1} &= \hat{\chi}_{\frac{q}{2},1} + \hat{\chi}_{\frac{q}{2},11} \\ \hat{\chi}^{SW}_{i,2} &= \hat{\chi}_{i,7} + \hat{\chi}_{i,13} + \hat{\chi}_{i,17} + \hat{\chi}_{i,23}, \quad 2 \le i < \tfrac{q}{2},\ i \in \mathcal{I}_2 \\ \hat{\chi}^{SW}_{\frac{q}{2},2} &= \hat{\chi}_{\frac{q}{2},7} + \hat{\chi}_{\frac{q}{2},13} \end{aligned} \tag{4.4}$$

The second class consists of the $\mathcal{SW}$-algebras existing for parabolic values of c. For these rational models we always have $\tilde{c} = \frac{3}{2}$. We divide this class into two series A and B. The first series A contains the algebras with non-vanishing self coupling constant. The central charge is given by $c = \frac{3}{2}(1 - 16k)$ with $4k \in \mathbb{N}$. The superconformal dimension of the additional generator is $\delta = 8k$. Realizations are provided for $k = \frac{1}{4}, \frac{1}{2}, \frac{3}{4}$. The algebras with vanishing self coupling constant belong to the second series B. Here we have $c = \frac{3}{2}(1 - 16k)$ with $2k \in \mathbb{N}$ and $\delta = 3k$. Realizations are present for $k = 1, \frac{3}{2}, 2$. The possible h-values can be parametrized with the following formula:

$$h_{r,r} = k(r^2 - 1), \quad h_{r,-r} = k(r^2 - 1) + \tfrac{1}{2} r^2 \tag{4.5}$$

The table below contains the h-values of the HWRs for both series. Note that in the Ramond sector one has to add $\frac{1}{16}$ to the values given in the table. For series A in the NS sector the value $h = 0$ appears twice, once with $w = 0$ (vacuum) and once with $w \neq 0$.

Furthermore we observed that the h-values $h_{0,0}$, $h_{1,1}$, $h_{1,-1}$ in the R sector are doubly degenerate. For series B only one representation is possible with $w^2 \neq 0$ in the NS sector, $h_{\frac{1}{2},\frac{1}{2}}$ $(k-[k]=0)$ or $h_{\frac{1}{2},-\frac{1}{2}}$ $(k-[k]=\frac{1}{2})$.

series		NS sector	R sector
A	$h_{\frac{l}{4k},\frac{l}{4k}}$	$l=0,\ldots,4k,8k$	$l=0,\ldots,4k$
$(4k\in\mathbb{N})$	$h_{\frac{l'}{4k+2},-\frac{l'}{4k+2}}$	$l'=1,\ldots,4k+1$	$l'=0,\ldots,4k+2$
B	$h_{\frac{l}{2k},\frac{l}{2k}}$	$l=0,\ldots,[k],2k$	$l=k-[k],\ldots,k$
$(2k\in\mathbb{N})$	$h_{\frac{l'}{2k+1},-\frac{l'}{2k+1}}$	$l'=0,\ldots,[k+\frac{1}{2}]$	$l'=\frac{1}{2}-(k-[k]),\ldots,k+\frac{1}{2}$

With the knowledge of this data M. Flohr was able to give the explicit form of the characters of the HWRs [18].

The third class is built by the $\mathcal{SW}(\frac{3}{2},\delta)$-algebras existing for $c=c(1,s),\ s\in 2\,\mathbb{N}+1$. In complete analogy the conformal case we obtain infinitely many HWRs lying on one parameter branches. The following table shows the functional dependence of w^2 and h:

$\mathcal{SW}(\frac{3}{2},\frac{5}{2}):\quad c=-\frac{5}{2}$		$\mathcal{SW}(\frac{3}{2},\frac{9}{2}):\quad c=-\frac{81}{10}$	
NS sector	R sector	NS sector	R sector
$\frac{h^2(6h+1)}{12}$	$\frac{(48h+5)(16h+1)^2}{24576}$	$-\frac{h^2(5h+2)(10h+3)^2}{7200}$	$-\frac{(16h+5)^2(80h+9)^2(80h+27)}{1887436800}$

These results support the assumption of a free field construction for this class of algebras in analogy to the conformal case [24].

Finally we discuss exceptional $\mathcal{SW}$-algebras. Here the knowledge of the possible HWRs is of special interest because it is a great help for the understanding of the structure of these algebras. Especially we are interested in new rational models with $\tilde{c}$ greater than $\frac{3}{2}$ and, as we have seen so far, exceptional algebras seem to be the only candidates.

At first we consider the cases $\mathcal{SW}(\frac{3}{2},4)$, $c=-13$ and $\mathcal{SW}(\frac{3}{2},6)$, $c=-18$, which have the same mixed structure of their HWRs as the values $c=-\frac{39}{4}$ and $c=-\frac{3}{2}$ of $\mathcal{SW}(\frac{3}{2},2)$. We stress that the conformal dimension of the additional generator is an even integer in these cases. Here are the concrete results ($q=w/C_{\phi\phi}^{\phi}$):

$\mathcal{SW}(\frac{3}{2},4):\quad c=-13$				$\mathcal{SW}(\frac{3}{2},6):\quad c=-18$	
NS sector		R sector		NS sector	
h	q	h	q	h	q
0	0	$\frac{1}{8}$	$\frac{63}{7904}$	0	0
$-\frac{1}{6}$	$\frac{1}{702}$	$-\frac{1}{24}$	$-\frac{947}{106704}$	$-\frac{1}{4}$	$\frac{17}{321024}$
$q=\frac{538h^2+549h+137}{10374}$		$p_0(h)\ q^2+p_1(h)\ q+p_2(h)=0$		$q=\frac{9076h^3+17404h^2+10827h+2169}{2588256}$	

with

$$p_0(h)=110202753024,\ p_1(h)=-11430322176h^2-8105330688h-1292683392$$
$$p_2(h)=296390656h^4+535662592h^3+360220288h^2+106998144h+11899017$$

Because of the rising complexity of the calculations we have not been able to calculate the restrictions in the R sector for the $\mathcal{SW}(\frac{3}{2},6)$. However, the results in the NS sector should be sufficient if one has more theoretical knowledge. So far there is no understanding of this mixed behaviour of the HWRs of these algebras.

For the remaining exceptional cases our calculations revealed the following new rational models. Because we do not list the w-values of the representations we point out that the h-values in the R-sector are degenerate in most cases. Furthermore we remind the reader of the possibility that some of the h-values may drop out by further investigation because our calculations yield only necessary conditions.

δ	c	$\tilde{c}$	sector	h
$\frac{7}{2}$	$-\frac{17}{11}$	$\frac{19}{11}$	NS	$0, \frac{1}{22}, \frac{7}{22}, \frac{13}{22}, \frac{2}{11}, \frac{25}{22}, \frac{3}{11}, -\frac{1}{11}, -\frac{1}{22}, -\frac{3}{22}$
			R	$\frac{3}{88}, \frac{19}{88}, \frac{27}{88}, \frac{51}{88}, \frac{59}{88}, \frac{67}{88}, \frac{123}{88}, \frac{131}{88}, \frac{25}{8}, -\frac{5}{88}$
4	$-\frac{185}{4}$	$\frac{7}{4}$	NS	$0, -2, -\frac{5}{4}, -\frac{7}{4}, -\frac{11}{6}, -\frac{13}{8}, -\frac{15}{8}, -\frac{19}{12}$
			R	$-\frac{45}{32}, -\frac{51}{32}, -\frac{61}{32}, -\frac{173}{96}, -\frac{185}{96}, -\frac{37}{32}, -\frac{47}{32}, -\frac{53}{32}$
4	$-\frac{120}{13}$	$\frac{24}{13}$	NS	$0, -\frac{11}{26}, -\frac{7}{26}, -\frac{5}{26}, -\frac{1}{26}, -\frac{6}{13}, -\frac{5}{13},$ $-\frac{4}{13}, -\frac{1}{13}, \frac{10}{13}, \frac{2}{13}, \frac{55}{26}, \frac{23}{26}, \frac{9}{26}, \frac{3}{26}$
			R	$4, \frac{20}{13}, \frac{18}{13}, \frac{17}{13}, \frac{12}{13}, \frac{6}{13}, \frac{3}{13}, \frac{2}{13},$ $\frac{1}{13}, -\frac{2}{13}, -\frac{3}{13}, -\frac{4}{13}, -\frac{5}{13}$
$\frac{11}{2}$	$-\frac{705}{8}$	$\frac{15}{8}$	NS	$0, -\frac{7}{2}, -\frac{11}{3}, -\frac{13}{4}, -\frac{15}{4}, -\frac{15}{8},$ $-\frac{23}{8}, -\frac{25}{8}, -\frac{29}{8}, -\frac{29}{12}, -\frac{79}{24}, -\frac{85}{24}$
			R	$-\frac{159}{64}, -\frac{191}{64}, -\frac{199}{64}, -\frac{211}{64}, -\frac{219}{64}, -\frac{227}{64},$ $-\frac{231}{64}, -\frac{235}{64}, -\frac{593}{192}, -\frac{605}{192}, -\frac{641}{192}, -\frac{701}{192}$
$\frac{11}{2}$	$-\frac{155}{19}$	$\frac{37}{19}$	NS	$0, -\frac{3}{19}, -\frac{5}{19}, -\frac{7}{19}, -\frac{8}{19}, -\frac{3}{38}, -\frac{5}{38}, -\frac{11}{38},$ $-\frac{13}{38}, -\frac{15}{38}, \frac{1}{19}, \frac{6}{19}, \frac{10}{19}, \frac{17}{19}, \frac{26}{19}, \frac{70}{19}, \frac{11}{38}, \frac{17}{38}, \frac{29}{38}, \frac{75}{38}$
6	$-\frac{2241}{20}$	$\frac{39}{20}$	NS	$0, -\frac{9}{2}, -\frac{14}{3}, -\frac{17}{4}, -\frac{19}{4}, -\frac{21}{5}, -\frac{22}{5},$ $-\frac{39}{10}, -\frac{47}{12}, -\frac{53}{20}, -\frac{89}{20}, -\frac{93}{20}, -\frac{137}{30}, -\frac{229}{60}$

In the case $\mathcal{SW}(\frac{3}{2},\frac{11}{2})$, $c = -\frac{155}{19}$ our calculations in the R sector yielded so far only one relation between h and q. Due to computational problems we have not been able to evaluate more complicated Jacobi identities and therefore the h-values of the model cannot be presented. As mentioned above the computational problems in the R sector were enormous for the $\mathcal{SW}(\frac{3}{2},6)$ and forced us to stop the calculations here.

5. Summary

In this paper we investigated the highest weight representations of the $N=1$ $\mathcal{SW}$-algebras with two generators for dimensions of the additional superprimary between two and six. For the algebras existing for discrete values of the central charge c we studied Jacobi identities. Our calculations yield necessary conditions for consistent highest weight representations. We stress the fact that in the minimal and parabolic cases the HWRs obtained this way coincide exactly with the expected values. This is a strong hint that in the other cases the given HWRs do indeed exist. For the generically existing algebra $\mathcal{SW}(\frac{3}{2},2)$ the Jacobi identities are trivial so that arbitrary HWRs are possible. Imposing the physical condition that null fields vanish in all HWRs we found for special c-values models fitting in the general pattern.

For the algebras existing for c-values contained in the minimal series of the super Virasoro algebra the allowed h-values are exactly those which are obtained if one assumes that the characters of the possible HWRs of the $\mathcal{SW}$-algebra are finite sums of superconformal characters and diagonalize the modular invariant partition function belonging to this c-value according to the ADE-classification. Furthermore we observed that in the cases where two different modular invariant partition functions are possible for a single c-value there indeed exist two different $\mathcal{SW}(\frac{3}{2},\delta)$-algebras diagonalizing both solutions of A. Cappelli et al. Looking at the partition functions one may conjecture that the algebra with the higher conformal dimension of the simple additional superprimary is a subalgebra of the other algebra existing for the special c-value. This fact has already been pointed out in [21][8]. Our results suggest that for every modular invariant partition function of A. Cappelli et al. there exists a $\mathcal{SW}(\frac{3}{2},\delta)$-algebra diagonalizing it.

As expected we obtained for the parabolic algebras rational models with $\tilde{c}=\frac{3}{2}$. We also gave a parametrization of the h-values. After the presence of this data the characters of the HWRs were given in [18]. For the $\mathcal{SW}(\frac{3}{2},\delta)$-algebras existing for $c=c(1,s)$ with $\delta=s-\frac{1}{2}$, $2\leq s\in\mathbb{N}$ our results propose infinitly many HWRs lying on one parameter branches in analogy to the conformal case [24][12].

For the $\mathcal{SW}(\frac{3}{2},\delta)$-algebras with $\delta\in\{2,4,6\}$ we found a new interesting structure of their HWRs for special c-values. Here one has a mixed behaviour: besides a continuous one parameter branch HWRs with discrete values of the highest weight exist.
The remaining exceptional algebras lead to new rational models with $2>\tilde{c}>\frac{3}{2}$. With this data the first step to a complete understanding of these cases is done.

Acknowledgements

We are very grateful to W. Nahm, R. Blumenhagen, M. Flohr, J. Kellendonk, S. Mallwitz, A. Recknagel, M. Rösgen, M. Terhoeven and R. Varnhagen for many useful discussions. It is a pleasure to thank the Max-Planck-Institut für Mathematik in Bonn-Beuel especially M. Auferkorte, Th. Berger and S. Maurmann because the greatest part of the calculations has been performed there.

References

[1] F.A. Bais, P. Bouwknegt, M. Surridge, K. Schoutens
Extensions of the Virasoro Algebra Constructed form Kac-Moody Algebras Using Higher Order Casimir Invariants, Nucl. Phys. **B304** (1988) p. 348

[2] F.A. Bais, P. Bouwknegt, M. Surridge, K. Schoutens
Coset Construction for Extended Virasoro Algebras
Nucl. Phys. **B304** (1988) p. 371

[3] J. Balog, L. Fehér, P. Forgács, L. O'Raifeartaigh, A. Wipf
Kac-Moody Realization of $\mathcal{W}$-Algebras
Phys. Lett. **B244** (1990) p. 435

[4] A.A. Belavin, A.M. Polyakov, A.B. Zamolodchikov
Infinite Conformal Symmetry in Two-Dimensional Quantum Field Theory
Nucl. Phys. **B241** (1984) p. 333

[5] M.A. Bershadsky, V.G. Knizhnik, M.G. Teitelman
Superconformal Symmetry in Two Dimensions
Phys. Lett. **B151** (1985) p. 31

[6] A. Bilal, J.L. Gervais
Systematic Construction of Conformal Theories with Higher-Spin Virasoro Symmetries, Nucl. Phys. **B318** (1989) p. 579

[7] R. Blumenhagen, *Covariant Construction of $N = 1$ Super $\mathcal{W}$-Algebras*
Preprint BONN-HE-91-20 (1991), to appear in Nucl. Phys. **B**

[8] R. Blumenhagen, W. Eholzer, A. Honecker, R. Hübel
New N=1 Extended Superconformal Algebras with Two and Three Generators
Preprint BONN-HE-92-02 (1992), to appear in Int. Jour. Mod. Phys. **A**

[9] R. Blumenhagen, M. Flohr, A. Kliem, W. Nahm, A. Recknagel, R. Varnhagen
$\mathcal{W}$-Algebras with Two and Three Generators, Nucl. Phys. **B361** (1991) p. 255

[10] P. Bouwknegt, *Extended Conformal Algebras*, Phys. Lett. **B207** (1988) p. 295

[11] A. Cappelli, C. Itzykson, J.B. Zuber
The A-D-E Classification of Minimal and $A_1^{(1)}$ Conformal Invariant Theories
Comm. Math. Phys. 113 (1987) p. 1

[12] W. Eholzer, M. Flohr, A. Honecker, R. Hübel, W. Nahm, R. Varnhagen
Representations of $\mathcal{W}$-Algebras with Two Generators and New Rational Models
Preprint BONN-HE-91-22 (1991), to appear in Nucl. Phys. **B**

[13] H. Eichenherr, *Minimal Operator Algebras in Superconformal Quantum Field Theory*
Phys. Lett. **B151** (1985) p. 26

[14] V.A. Fateev, S.L. Lyk'anov
The Models of Two Dimensional Quantum Conformal Field Theory with $\mathbb{Z}_n$ Symmetry
Int. Journ. Mod. Phys. **A3** (1988) p. 507

[15] V.A. Fateev, A.B. Zamolodchikov
Conformal Quantum Field Theory Models in Two Dimensions Having $\mathbb{Z}_3$ Symmetry, Nucl. Phys. **B280** (1987) p. 644

[16] J.M. Figueroa-O'Farrill, S. Schrans, *The Spin 6 Extended Conformal Algebra*
Phys. Lett. **B245** (1990) p. 471

[17] J.M. Figueroa-O'Farrill, S. Schrans, *The Conformal Bootstrap and Super $\mathcal{W}$ Algebras*
Int. Jour. Mod. Phys. **A7** (1992) p. 591

[18] M. Flohr, *$\mathcal{W}$-Algebras, New Rational Models and Completeness of the $c = 1$ Classification*
Preprint BONN-HE-92-08 (1992)

[19] P. Goddard, A. Kent, D. Olive
Unitary Representations of the Virasoro and Super-Virasoro Algebra
Comm. Math. Phys. 103 (1986) p. 105

[20] P. Goddard, A. Kent, D. Olive
Virasoro Algebras and Coset Space Models
Phys. Lett. **B152** (1985) p. 88

[21] K. Hornfeck, *Realizations for the $\mathcal{SW}(\frac{7}{2})$-Algebra*

and the Minimal Supersymmetric Extension of $\mathcal{WA}_3$
Preprint London, King's College 91.08.26
[22] R. Hübel, *Darstellungstheorie von* $\mathcal{W}$*- und Super-*$\mathcal{W}$*-Algebren*
Diplomarbeit BONN-IR-92-11 (1992)
[23] T. Inami, Y. Matsuo, I. Yamanaka
Extended Conformal Algebras with $N = 1$ *Supersymmetry*
Phys. Lett. **B215** (1988) p. 701
[24] H.G. Kausch
Extended Conformal Algebras Generated by a Multiplet of Primary Fields
Phys. Lett. **B259** (1991) p. 448
[25] H.G. Kausch, G.M.T. Watts, *A Study of* $\mathcal{W}$*-Algebras Using Jacobi Identities*
Nucl. Phys. **B354** (1991) p. 740
[26] S. Komata, K. Mohri, H. Nohara
Classical and Quantum Extended Superconformal Algebra
Nucl. Phys. **B359** (1991) p. 168
[27] G. Mussardo, G. Sotkov, M. Stanishkov
Ramond Sector of Supersymmetric Minimal Models
Phys. Lett. **B195** (1987) p. 397
[28] G. Mussardo, G. Sotkov, M. Stanishkov
Fine Structure of Supersymmetric Operator Product Expansion Algebras
Nucl. Phys. **B305** (1988) p. 69
[29] W. Nahm, *Chiral Algebras of Two-Dimensional Chiral Field Theories and Their Normal Ordered Products* , Proceedings Trieste Conference on
Recent Developments in Conformal Field Theories, ICTP, Trieste (1989) p. 81
[30] M. Scheunert, *The Theory of Lie Superalgebras*
Lecture Notes in Mathematics 716 (1979)
[31] R. Varnhagen, *Characters and Representations of New Fermionic* $\mathcal{W}$*-Algebras*
Phys. Lett. **B275** (1992) p. 87
[32] A.B. Zamolodchikov
Infinite Additional Symmetries in Two-Dimensional Conformal Quantum Field Theory, Theor. Math. Phys. 65 (1986) p. 1205